Massachusetts Institute of Technology
1.200J—Transportation Systems Analysis: Performance and Optimization
Fall 2015 — TA: Wichinpong "Park" Sinchaisri

Recitation 4
Unit 2 — Optimization Methodology

1 IP: Branch and Bound

$$\begin{aligned} \underset{x_1,x_2}{\text{Maximize}} \quad & Z = -x_1 + 4x_2 \\ \text{subject to} \quad & -10x_1 + 20x_2 \leq 22, \\ & 5x_1 + 10x_2 \leq 49, \\ & x_1 \leq 5, \\ & x_1, x_2 \geq 0. \\ & x_1, x_2 \text{ integer.} \end{aligned}$$

2 IP: Formulations

2.1 Dream Team

We are selecting 12 students for the upcoming International Transportation Trivia Tournament from a short list of 20 students: $p_1, ..., p_{20}$.

- For each student, we have collected several statistics: r_i risk-taking skill, a_i applied mathematics background, h_i height, s_i class score, and d_i deductive reasoning skill.
- The 20 students have been divided to 4 broad categories: fast buzzer (FB) $(p_1, .., p_5)$, mental supporter (MS) $(p_4, ..., p_{11})$, transportation guru (TG) $(p_9, ..., p_{16})$, and entertainers (E) $(p_{16}, ..., p_{20})$. Notice that there are students that can be multiple roles. For example, Student 4 can be used both as a fast buzzer and a mental supporter.
- Students 4, 8, 15, 20 are undergraduate students, while all of the rest are graduate students.
- Students 1, 7, 12, 16 are from Antarctica. The rest are from different regions in/outside this world.

Formulate an IP to maximize the average class score with the following constraints:

(a) For balance purpose, the team should have at least 3 fast buzzers, 4 mental supporters, 4 transportation gurus, and 3 entertainers, which implies that some students with dual roles should be selected.

(b) At least 2 undergraduate students should be selected.

(c) The mean for each statistics should be at least R, A, H, S, D, respectively.

(d) Student 5 declares: "I will not join the team if Student 9 is selected!"

(e) Students 2 and 19 are dating and can never be separated, ever.

(f) For diversity reason, we cannot select more than 3 from the same region.

2.2 Aircraft Painting

With over 30 years of experience in painting and protecting aircraft from corrosion, JetSplash ensures the highest quality painting standards through its state-of-the-art three-step painting operations. Each batch of aircrafts must be painted in the order: first, second, third painting station. JetSplash must color five batches of aircrafts of different types. Painting batch i at station j takes a time s_{ij} expressed in hours in the matrix below:

$$\begin{pmatrix} 3 & 1 & 1 \\ 2 & 1.5 & 1.5 \\ 3 & 1.2 & 1.3 \\ 2 & 2 & 2 \\ 2.1 & 2 & 3 \end{pmatrix}$$

Formulate a mathematical program for JetSplash to schedule its painting operations at the stations so that the ending time of the last batch is minimized.