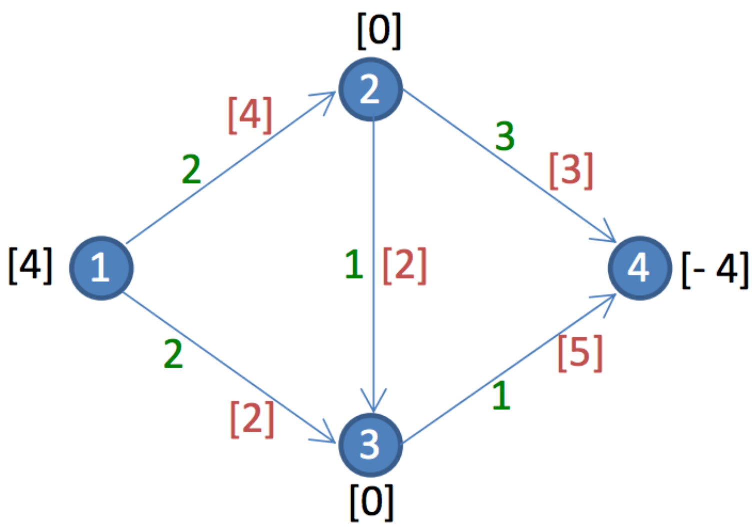
Massachusetts Institute of Technology
1.200J—Transportation Systems Analysis: Performance and Optimization
Fall 2015 — TA: Wichinpong "Park" Sinchaisri

Recitation 10
Unit 4 — Network Flows: Models and Algorithms

1 Minimum Cost Flow Problems



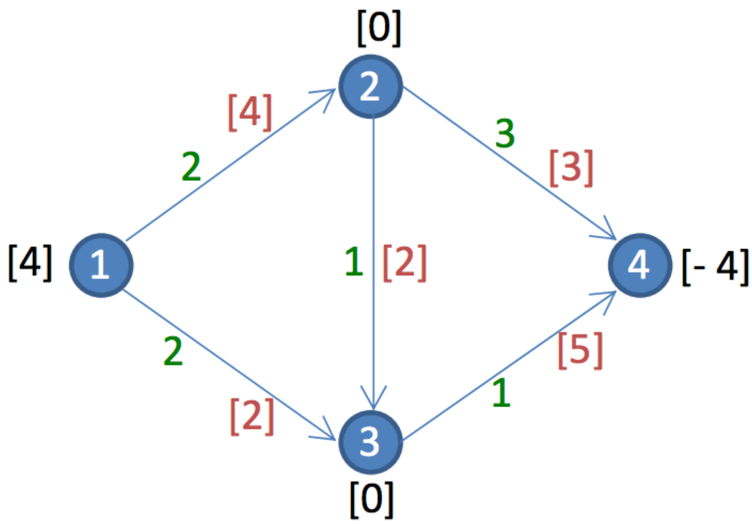
1.1 Cycle-Cancelling

(i) Start with a feasible flow f.

(ii) Find a negative cost cycle in the residual graph $G(f)$.

(iii) If so, augment the flow along the cycle, decrease the cost of the flow, and repeat the procedure.

(iv) When no such cycle exists, we obtain the optimal flow. Convert back to the original graph.



1.2 Successive Shortest Paths

(i) Pseudoflow f: set $f = 0, \pi = 0$ and $e(u) = b_u$ for all $u \in N$.

(ii) While there exists some excess node:

- Select node u with excess supply, node v with deficit demand.
- Find the shortest path P from u to v. (Dijkstra's)
- Update the potentials $\pi'_v = \pi_v - d(u, v)$.
- Send flow from u to v along P with value $\min\{c_f(P), e(u), -e(v)\}$.
- Update reduced costs $c'_{ij} = c_{ij} + d(i) - d(j)$ and the residual network.

(iii) Terminate when we get feasible solution. All reduced costs along the shortest paths are 0's.