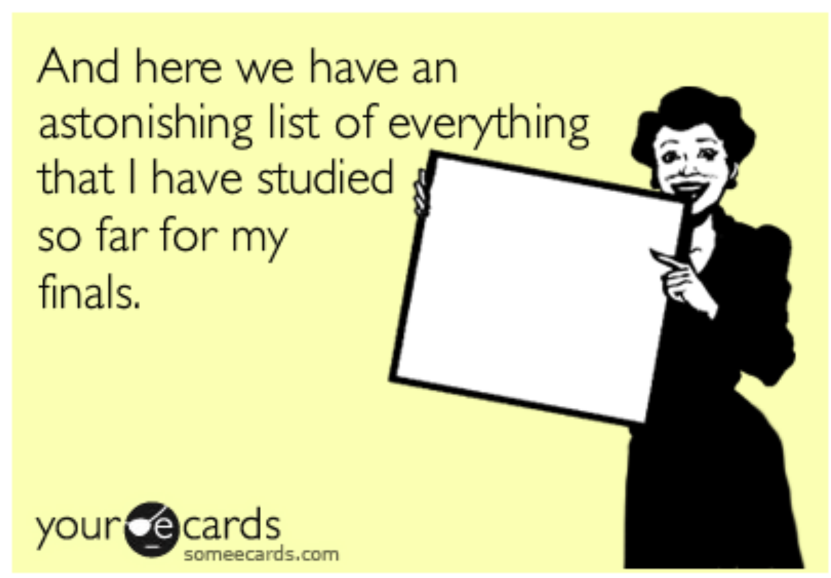
Welcome to DMD Recitation 7! ☺

- Due electronically on Tuesday May 26, 2015 at 11:59PM EDT
  - Endurance case, as a team, no memo necessary
  - Submit the PDF and accompanying Excel files on Stellar.
- To reduce background noise, please mute your phone/ computer!
- Please feel free to raise your hand or chat through WebEx if you have any questions or comments!

Outline

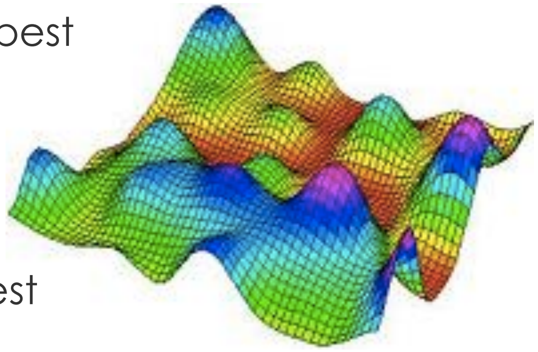
- Quick Endurance clarifications
- Optimization Examples
  - Summary of different techniques
  - Example 1: Job Assignment
  - Example 2: Airline Pricing



Endurance Clarifications

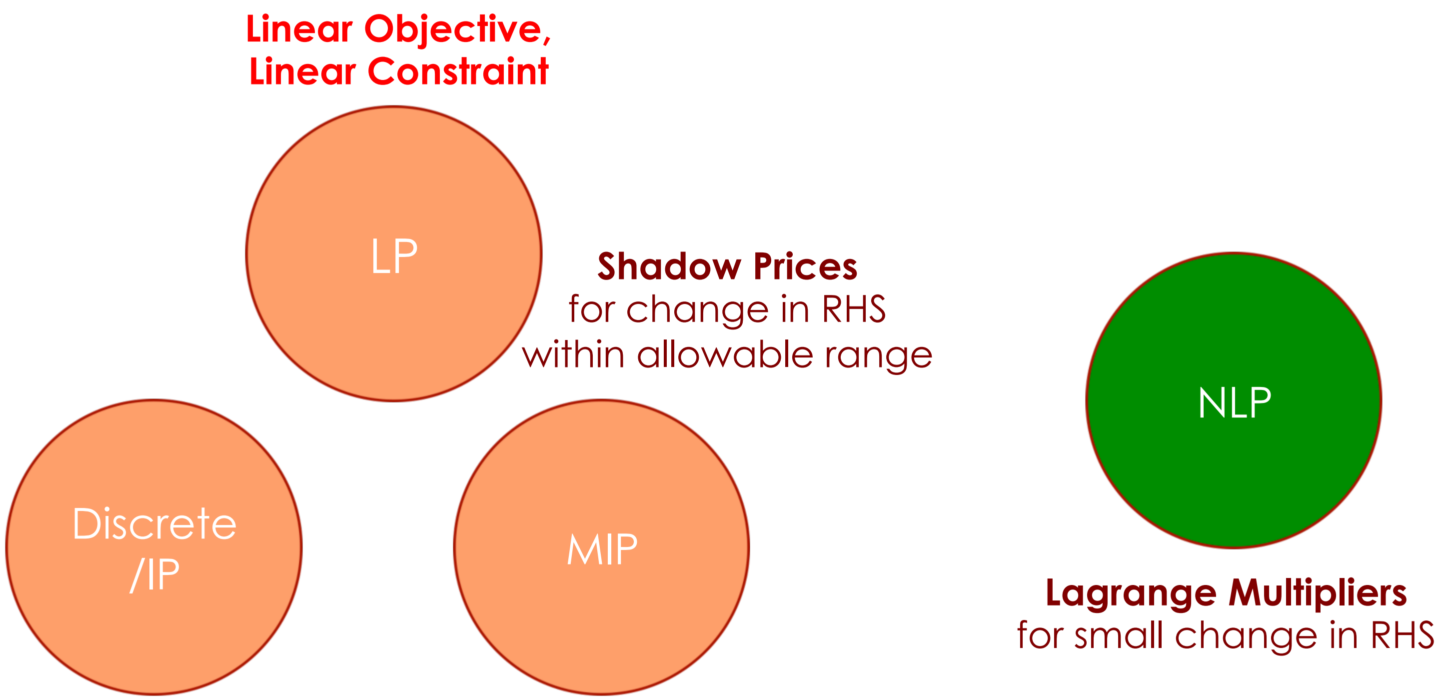
- It is possible to get **different solutions** when you run the solver more than once, or on different computers for the same formulation.

- Excel GRG Nonlinear solver runs the following algorithm:
  - Step 1. **Choose a random starting point**
  - Step 2. Search in the neighborhood of starting point for best LOCAL solution
  - Step 3. Once found, output the locally optimal solution.



- 2 options:
  - Manually run the model several times and record the best solution.
  - Click on "Options" in the Excel Solver, go the the tab "GRG Nonlinear" and check "MultiStart".

Which kind of optimization?



Solver: Simplex LP

Solver: GRG Nonlinear

Example 1: Job Assignment

- You are the manager at BDM corporation and your current assignment is to allocate each of 6 employees to one of 4 tasks
- Not every employee is qualified for each task
- At most 2 employees per task!

Qualification	Task A	Task B	Task C	Task D
Employee 1	Yes	Yes	Yes	No
Employee 2	No	Yes	Yes	No
Employee 3	Yes	No	Yes	Yes
Employee 4	Yes	Yes	No	Yes
Employee 5	Yes	No	No	Yes
Employee 6	Yes	Yes	Yes	Yes

Example

- Each employee has a different level of expertise in each task, and therefore their productivity levels differ for each task.

- Productivity levels as units of jobs completed per hour

Productivity	Task A	Task B	Task C	Task D
Employee 1	1	2	1	0
Employee 2	0	3	1	0
Employee 3	3	0	2	3
Employee 4	2	3	0	2
Employee 5	2	0	0	2
Employee 6	1	2	2	3

Step 1. Decision Variables

- X1A = 1 if employee 1 is selected for task A
- Employee 1:
  - X1A, X1B, X1C

Qualification	Task A	Task B	Task C	Task D
Employee 1	Yes	Yes	Yes	No
Employee 2	No	Yes	Yes	No
Employee 3	Yes	No	Yes	Yes
Employee 4	Yes	Yes	No	Yes
Employee 5	Yes	No	No	Yes
Employee 6	Yes	Yes	Yes	Yes

- X1A, X1B, X1C
- X2B, X2C
- X3A, X3C, X3D
- X4A, X4B, X4D
- Etc.

Qualification	Task A	Task B	Task C	Task D
Employee 1	Yes	Yes	Yes	No
Employee 2	No	Yes	Yes	No
Employee 3	Yes	No	Yes	Yes
Employee 4	Yes	Yes	No	Yes
Employee 5	Yes	No	No	Yes
Employee 6	Yes	Yes	Yes	Yes

Step 2. Objective Function

- **Maximize productivity** = total number of jobs completed per hour across all employees
- Productivity of employee 1:
  - 1*X1A + 2*X1B + 1*X1C

Productivity	Task A	Task B	Task C	Task D
Employee 1	1	2	1	0
Employee 2	0	3	1	0
Employee 3	3	0	2	3
Employee 4	2	3	0	2
Employee 5	2	0	0	2
Employee 6	1	2	2	3

Step 3. Constraints

- Each task must be assigned to at least one employee
  - **X1A + X3A + X4A + X5A + X6A >= 1**

Qualification	Task A	Task B	Task C	Task D
Employee 1	Yes	Yes	Yes	No
Employee 2	No	Yes	Yes	No
Employee 3	Yes	No	Yes	Yes
Employee 4	Yes	Yes	No	Yes
Employee 5	Yes	No	No	Yes
Employee 6	Yes	Yes	Yes	Yes

Step 3. Constraints

- For each task, no more than 2 employees
  - **X1B + X2B + X4B + X6B <= 2**

Qualification	Task A	Task B	Task C	Task D
Employee 1	Yes	Yes	Yes	No
Employee 2	No	Yes	Yes	No
Employee 3	Yes	No	Yes	Yes
Employee 4	Yes	Yes	No	Yes
Employee 5	Yes	No	No	Yes
Employee 6	Yes	Yes	Yes	Yes

Step 3. Constraints

- Each employee needs to be assigned exactly one task
  - **X3A + X3C + X3D = 1**

Qualification	Task A	Task B	Task C	Task D
Employee 1	Yes	Yes	Yes	No
Employee 2	No	Yes	Yes	No
Employee 3	Yes	No	Yes	Yes
Employee 4	Yes	Yes	No	Yes
Employee 5	Yes	No	No	Yes
Employee 6	Yes	Yes	Yes	Yes

Step 3. Constraints

- Employee 1 is a novice at Task A and therefore can only be assigned to it if the more experienced employee 3 was assigned to it
  - **X1A <= X3A**

Step 3. Constraints

- Employees 4 and 6 do not work well together and therefore cannot be assigned to the same task
  - **X4A + X6A <= 1**
  - **X4B + X6B <= 1**
  - **X4C + X6C <= 1**
  - **X4D + X6C <= 1**

Step 3. Constraints

- Employees 2 and 3 do work well together. If they are assigned to the same task, then their combined productivity on that task enjoys an extra boost of one unit, in addition to their individual productivities

- Need to add new binary variables!
  - **YA** = 1 if employees 2 and 3 are assigned together for task A
  - **YB, YC, YD**

Step 3. Constraints

- Employees 2 and 3 do work well together. If they are assigned to the same task, then their combined productivity on that task enjoys an extra boost of one unit, in addition to their individual productivities

  - **What constraints to add?**
  - Suppose 2 and 3 assigned both to task A, then
    - **X2A + X3A >= 2** **(YA=1)**
  - If not, then it does not really matter!
    - **X2A + X3A >= 0** **(YA=0)**

X2A + X3A >= 2YA

You may think when YA = 0, both X2A and X3A can be 1 too?
When that's the case, Solver will pick YA = 1 to maximize productivity!

Step 3. Constraints

- Employees 2 and 3 do work well together. If they are assigned to the same task, then their combined productivity on that task enjoys an extra boost of one unit, in addition to their individual productivities

  - **What constraints to add?**
    - **X2A + X3A >= 2YA**
    - **X2B + X3B >= 2YB**
    - **X2C + X3C >= 2YC**
    - **X2D + X3D >= 2YD**

Step 3. Constraints

- Employees 2 and 3 do work well together. If they are assigned to the same task, then their combined productivity on that task enjoys an extra boost of one unit, in addition to their individual productivities

  - **What also do we need to change?**
  - **How to change the objective function?**
  - Add the following terms
    - **… + YA + YB + YC + YD**

Takeaways: Discrete Optimization

- Decision variables take discrete values
- Tying binary v's to remaining v's
  - It is important to illustrate the logical relationship between the binary and other decision variables.
  - Ex: If a facility is closed, then we cannot produce any items at that facility.
- Excel Solver
  - Constraint: set the cells in "int" or "bin"
  - Make sure we select them as part of "Changing Variable Cells" prior to inputting the constraints
  - RHS cells must only contain constants.

Example 2: Airline Pricing

- MBA students travel a lot.
- Sloan and HBS have purchased one 120-seat plane each.
- The two airlines are competing for customers, since the price-sensitive students choose the lowest fare.
- SloanAir is run by former DMD students who decided to use optimization to maximize the airline revenue.
- Meanwhile, HBS students who run HBSAir just use a "sensible" price: $p_2 = \$50$
- A customer survey has shown that the demand for travel for SloanAir is:

$$D_1 = 120 - 0.9\, p_1 + 0.5\, p$$

1. Define the decision variables

- Only 1 decision variable in this case:
  - p_1 = price charged per ticket on SloanAir

- HBSAir's price p_2 is fixed at \$50

2. Write the objective function

- We are trying to maximize SloanAir's revenue

- Revenue = price * quantity = $p_1 * D_1$

- max $p_1 * (\overbrace{120 - 0.9\ p_1 + 0.5\ p_2}^{\boxed{145 - 0.9\ p_1}})$

- max $145p_1 - 0.9p_1^2$

3. Write down the constraints

- Demand cannot exceed capacity (no overbooking):
  - D_1 <= 120
  - $145 - 0.9p_1$ <= 120

- Demand is nonnegative:
  - D_1 >= 0
  - $145 - 0.9p_1$ >= 0

4. Additional restrictions on the decision variables

- Only decision variable is price
- Should be nonnegative:
  - $p_1 >= 0$

Complete NLP model

max $\quad 145p_1 - 0.9p_1^2$

s.t. $\quad 145 - 0.9p_1 <= 120$

$145 - 0.9p_1 >= 0$

$p_1 >= 0$

Solution

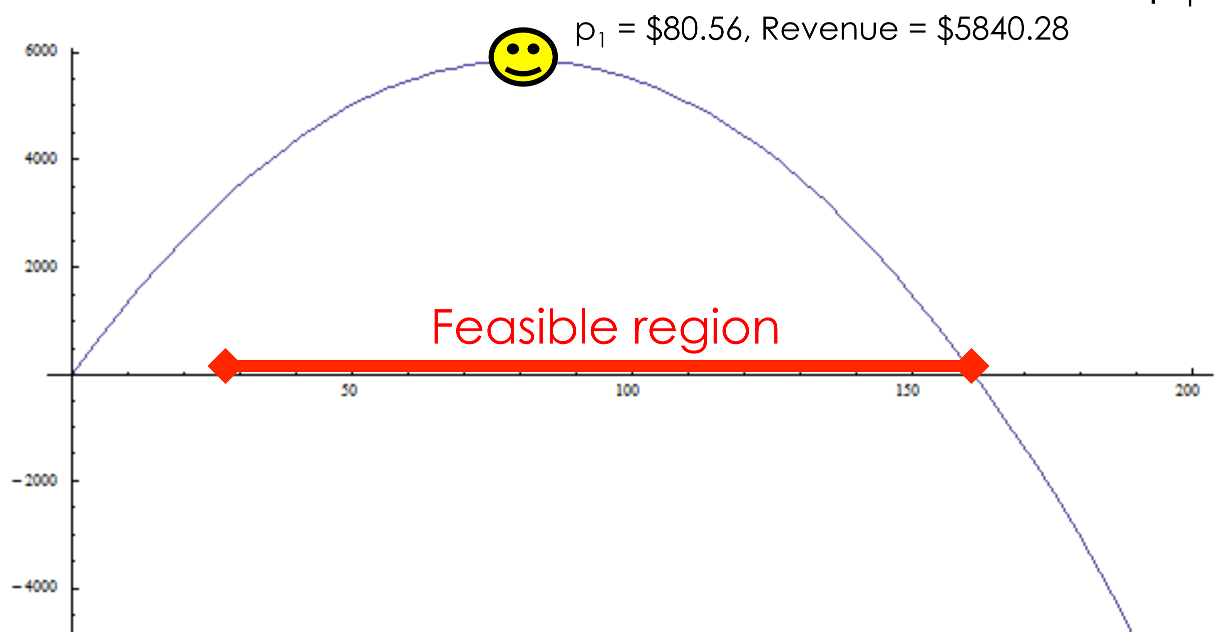
SloanAir price	80.55556	Capacity	120
HBSAir price	50		
Demand	72.5		
Objective function (revenue)	5840.278		

Lagrange multipliers will be 0
(The capacity constraint is not binding!)

Visualizing the solution

- We can use the constraints to find the feasible region
  - $145 - 0.9p_1 <= 120$ means $p_1 >= 27.78$
  - $145 - 0.9p_1 >= 0$ means $p_1 <= 161.11$

$$\begin{array}{ll} \max & 145p_1 - 0.9p_1^2 \\ \text{s.t.} & 145 - 0.9p_1 <= 120 \\ & 145 - 0.9p_1 >= 0 \\ & p_1 >= 0 \end{array}$$



Best Response to HBS prices

- HBS decides to experiment with prices, and sets the following prices on various days:

$$p_2 = 10,\ p_2 = 20,\ p_2 = 30,\ \ldots\ ,\ p_2 = 90,\ p_2 = 100$$

- For each choice of HBS's price, we can use Solver to find Sloan's optimal price: "best response"

Takeaways: Nonlinear Optimization

- Local Optimal Solutions
  - Excel Solver starts randomly at a point – find locally optimal solution.
  - Run Solver multiple times and record best solution!
- Sensitivity Analysis
  - No shadow prices here
  - Lagrange multipliers tells where the solution is located.
  - All LM's = 0 -> solution is at the interior of the feasible Region. All constraints are non-binding.
- Efficient Frontier
  - Changing RHS -> resolve and plot optimal objectives for various RHS values
- Choice of the Objective Functions
  - Try variations: minimize the sum, minimize the max, etc.

Wrap up! ☺

- Tonight, we covered more optimization examples.
- Due electronically on Tuesday May 26, 2015 at 11:59PM EDT
  - **Endurance Case** (Chap 8), as a team, no memo necessary
  - Submit the PDF and accompanying Excel files on Stellar.
- Last Weekend May 29/30:
  - **MTM Case** - please read and be prepared to discuss in class.
- Please email us any questions or comments/suggestions for improving the recitation.
- Office Hours now!