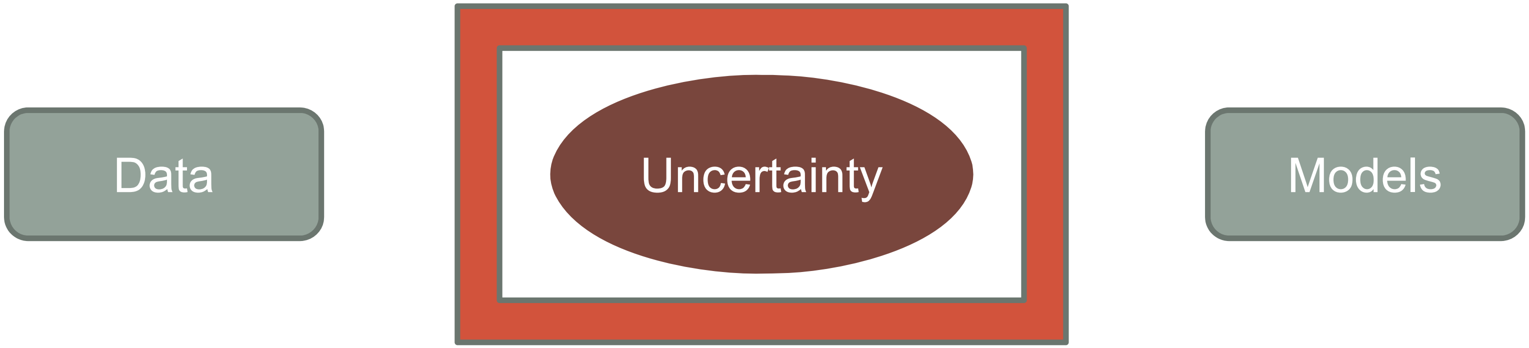
Welcome to DMD Recitation 1!

- Due electronically on Monday February 2, 2015
  - Airline on Ground (AOG) case – work with your team!
  - Exercises 2.13 and 2.30 – complete individually!
  - Submit the PDF files on Stellar.
- Google Doc for Teams: bit.ly/DMD16-Teams
- To reduce background noise, please mute your phone/ computer!
- Please feel free to raise your hand or chat through WebEx if you have any questions or comments!

Data, Models, and Decisions

- Decision making is hard due to uncertainty
- Use data and build models to make more informed decisions



Outline

- Decision Analysis
  - Tree construction
  - EMV calculations
  - Sensitivity Analysis

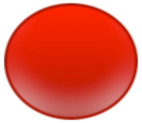
- Binomial Distribution
  - Understand the formula
  - Compute probabilities

Decision Analysis

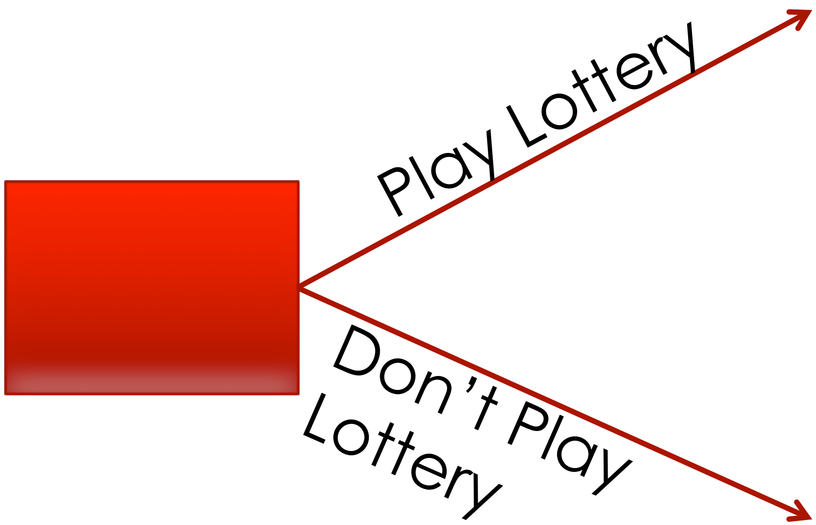
- Decision Tree
  - Logical and systematic way of organizing and representing various decisions and uncertainties

- Two types of nodes
  - **Decision node:** point where we can choose between alternatives

  - **Event node:** point with uncertainty that we cannot control

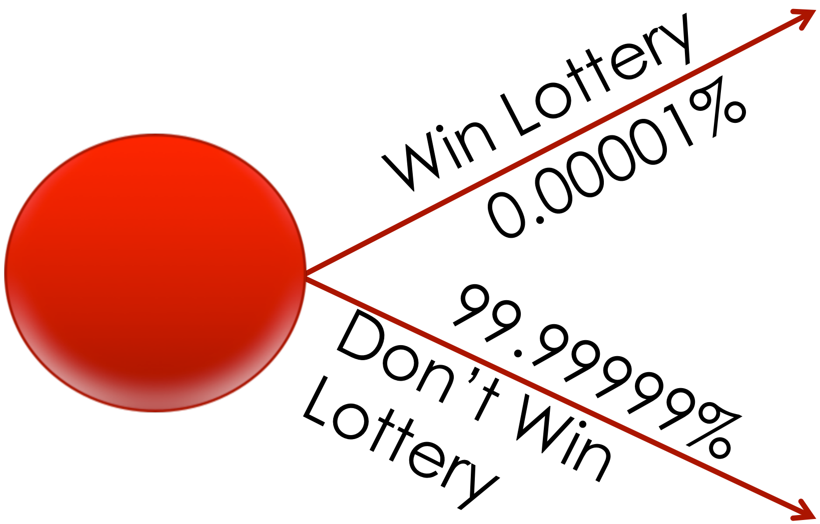


Examples of Nodes



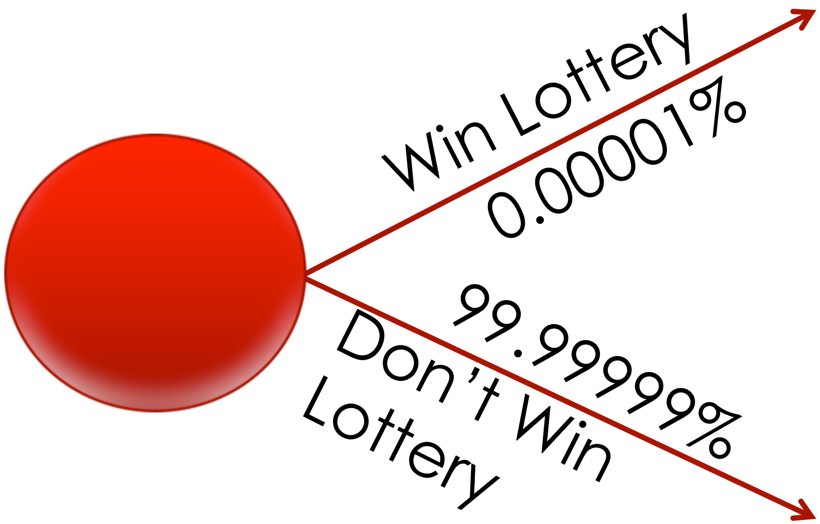
We have control!

Examples of Nodes



Outcome is Random!

Event Node Branches



- **Mutually Exclusive:**

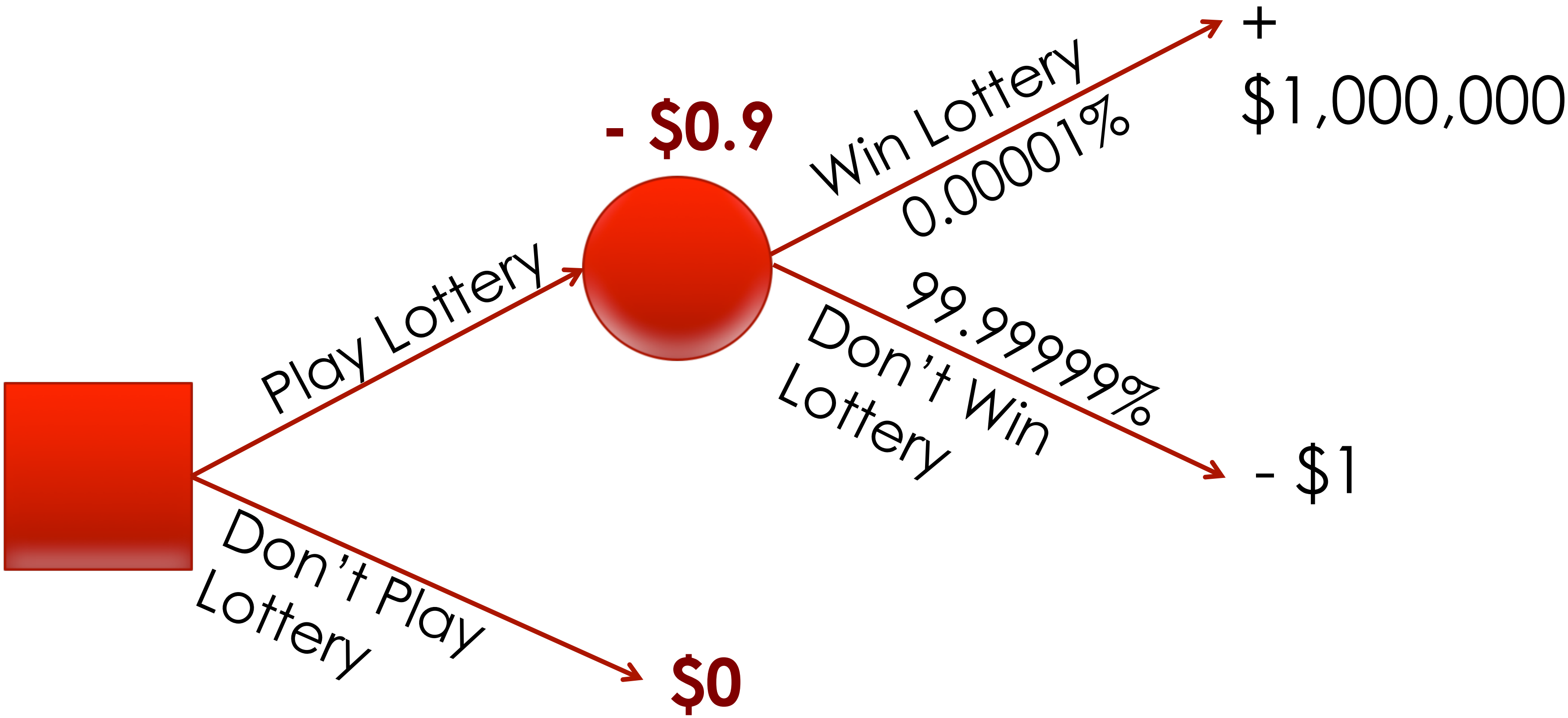
No two outcomes can happen simultaneously

- **Collectively Exhaustive:**

Set of all possible outcomes represents the entire range of possible outcomes

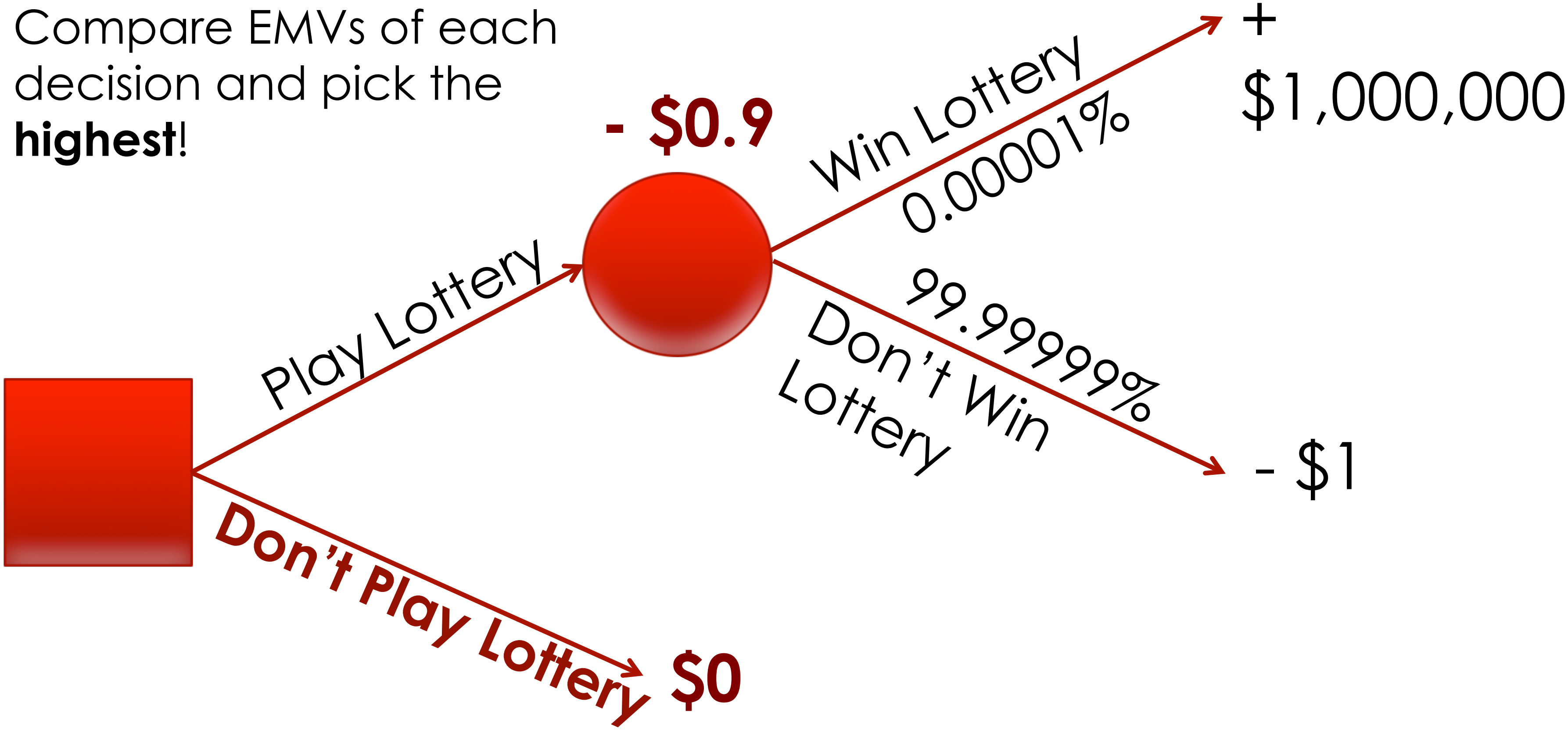
Sum of probabilities equals ONE

Should We Play the Lottery?



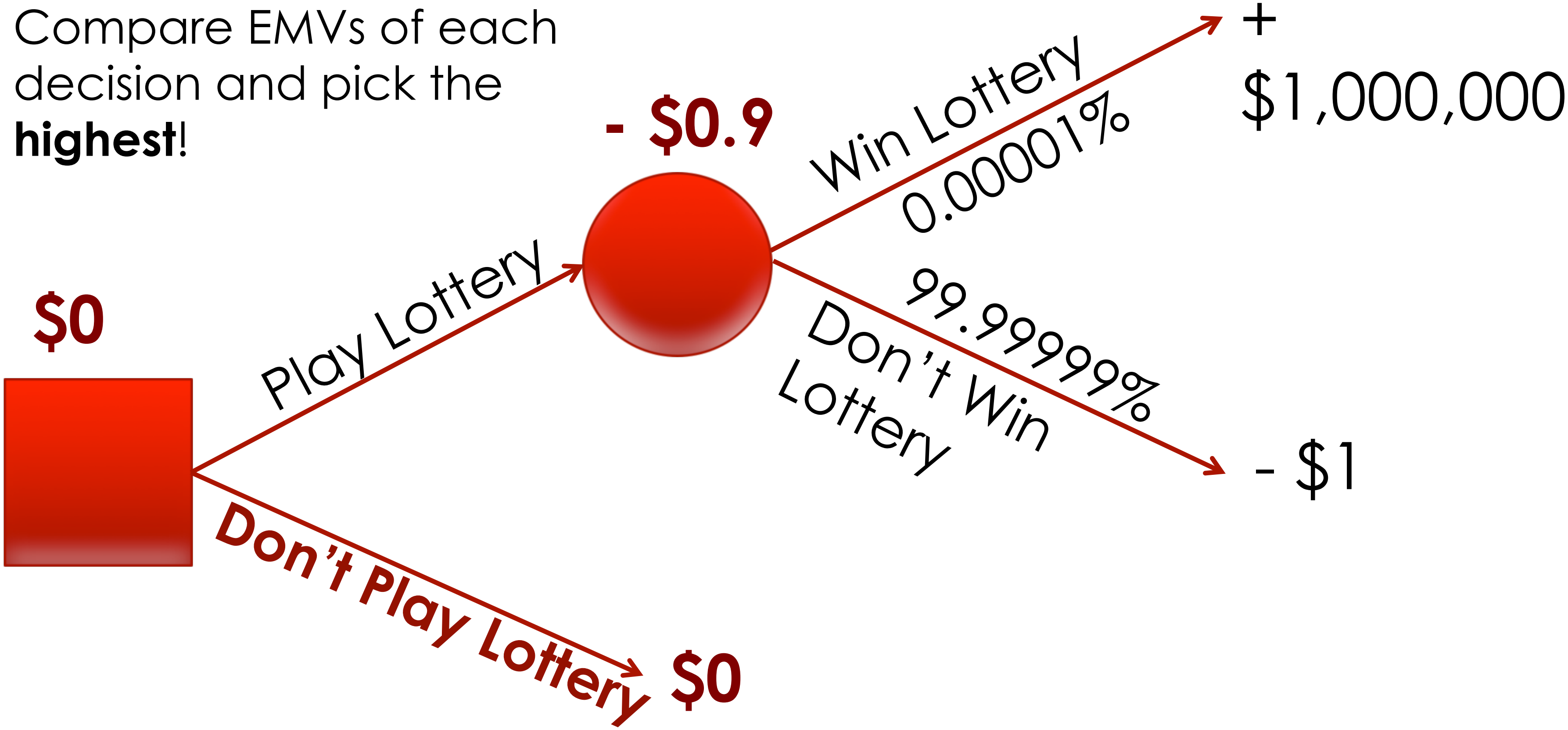
Should We Play the Lottery?

Compare EMVs of each decision and pick the **highest**!



Should We Play the Lottery?

Compare EMVs of each decision and pick the **highest**!



Decision Analysis Procedure

- List choices (decision nodes)
- List uncertain events (event nodes)
- Construct a decision tree
- Determine the probabilities of each outcome
- Determine the numerical values of the endpoints
- Solve using backward induction
  - Event nodes: calculate EMV
  - Decision nodes: choose decision with highest EMV
- Perform sensitivity analysis (What-if scenarios)

Kendall Crab and Lobster Case

- Shortly before noon, Jeff Daniels, director of Overnight Delivery Operations at Kendall Crab and Lobster (KCL) watched the weather channel:
  - Weather forecast predicted 50% chance that the storm hits Boston around 5pm

- With the chance of Logan airport closing, business travelers were also nervously awaiting further weather information
  - In the past, if a storm of this magnitude hits, 1 in 5 come with strong winds that force Logan to close

Operations

- Customers can order lobsters for next-day delivery prior to 5pm on day before delivery
  - Typical daily order of 3,000 lobsters
  - At 5:30pm, trucks from United Express pick up the lobsters and truck them to Logan airport
  - At 6:30pm, packed lobsters are flown to a processing and distributing facility in DC
  - By 10:30am of next day, lobsters are delivered

Earnings and Refund Policy

- Price charged to customers is $30/lobster, which includes all transportation costs
- When KCL ships a lobster via United Express, its unit contribution to earnings is $10/lobster
- If KCL cannot deliver the lobsters to customers, its policy is to give each customer a $20 discount coupon per lobster
  - Market research has shown that ~70% of the customers only redeem the coupons

Changes to Operations Due to Weather

- Rely on the Massachusetts Air Freight (MAF) which operates 50 miles away from Boston
  - If contacted before 5:30pm, MAF will pick up the lobsters from KCL and deliver them to an airport in Worcester to fly them to DC
  - Additional transportation cost of using MAF is $13/lobster in roughly 67% of the time, $19/lobster in the remaining 33%

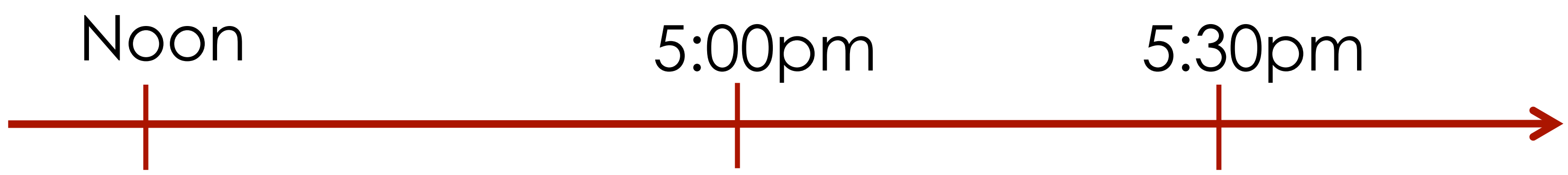
Changes to Operations Due to Weather

- Cancel orders and issue coupons
  - If the lobsters are not packaged yet, the incremental cost of cancelling the orders is ~$1/lobster
  - If lobsters were already packages, incremental cost is ~$1.25/lobster

- Deliver lobsters by truck to DC via the Eastern Parcel Delivery (EPD)
  - Arrangement needs to be made by noon!
  - Cost is $4/lobster 50% of the time, $3/lobster 25% of the time and $2/lobster 25% of the time.

What is the Best Decision?

- List Decisions that can be made over time



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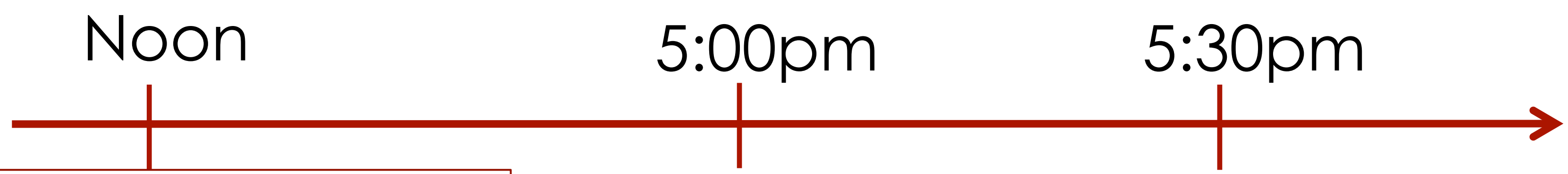
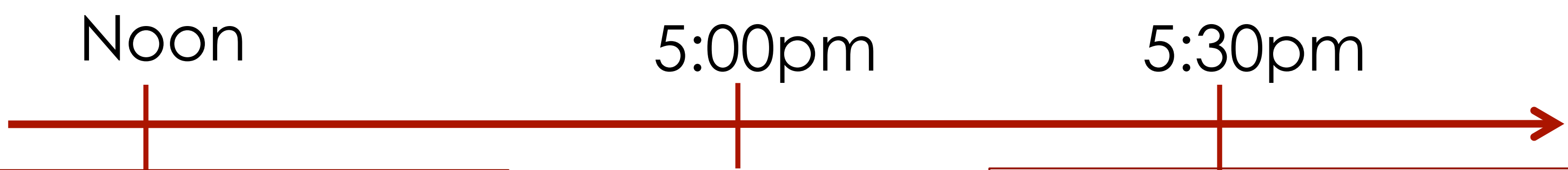


1. Use EPD to deliver by land
2. Wait until 5:00pm
3. Cancel orders and issue coupons

What is the Best Decision?

- List Decisions that can be made over time



Noon:

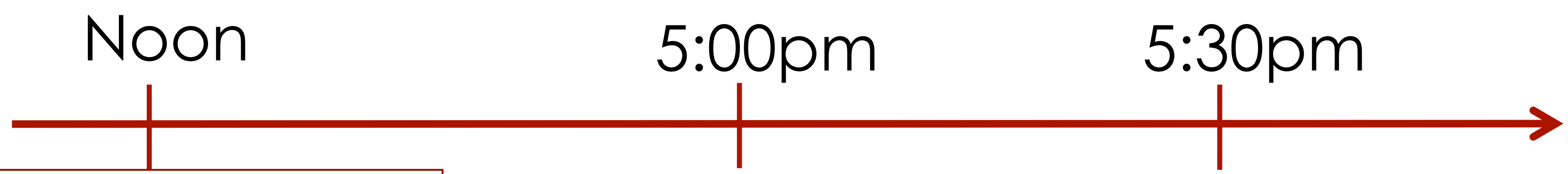
1. Use EPD to deliver by land
2. Wait until 5:00pm
3. Cancel orders and issue coupons

5:30pm:

1. USE MAF to deliver to Worcester
2. Cancel orders and issue coupons

What is the Best Decision?

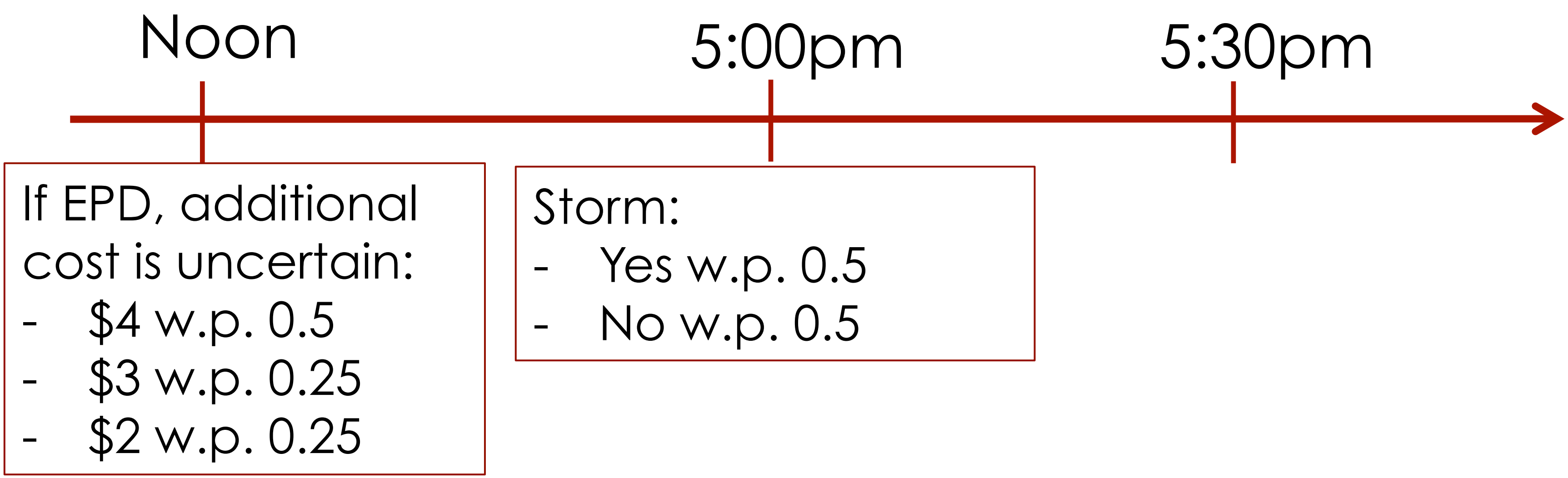
- List Uncertainties and their probability of occurrence



If EPD, additional cost is uncertain:
- $4 w.p. 0.5
- $3 w.p. 0.25
- $2 w.p. 0.25

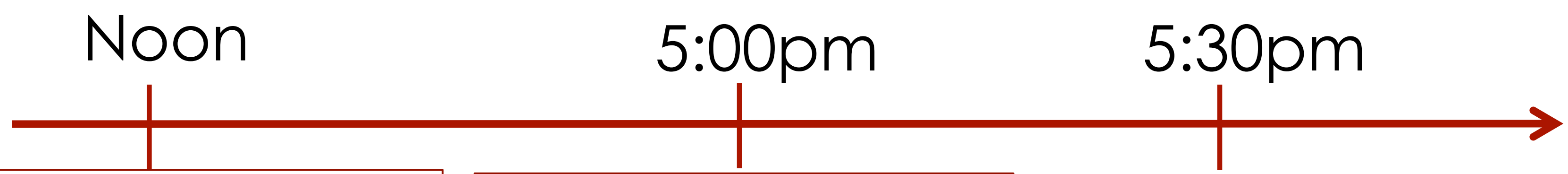
What is the Best Decision?

- List Uncertainties and their probability of occurrence



What is the Best Decision?

- List Uncertainties and their probability of occurrence



If EPD, additional cost is uncertain:
- $4 w.p. 0.5
- $3 w.p. 0.25
- $2 w.p. 0.25

Storm:
- Yes w.p. 0.5
- No w.p. 0.5

Logan Airport:
- Closed w.p. 0.2
- Open w.p. 0.8

What is the Best Decision?

- List Uncertainties and their probability of occurrence

Noon — 5:00pm — 5:30pm

Noon

If EPD, additional cost is uncertain:
- $4 w.p. 0.5
- $3 w.p. 0.25
- $2 w.p. 0.25

5:00pm

Storm:
- Yes w.p. 0.5
- No w.p. 0.5

Logan Airport:
- Closed w.p. 0.2
- Open w.p. 0.8

5:30pm

If MAF, cost is uncertain:
- $13 w.p. 0.67
- $19 w.p. 0.33

What is the Best Decision?

- List Uncertainties and their probability of occurrence

Noon

If EPD, additional cost is uncertain:
- $4 w.p. 0.5
- $3 w.p. 0.25
- $2 w.p. 0.25

5:00pm

Storm:
- Yes w.p. 0.5
- No w.p. 0.5

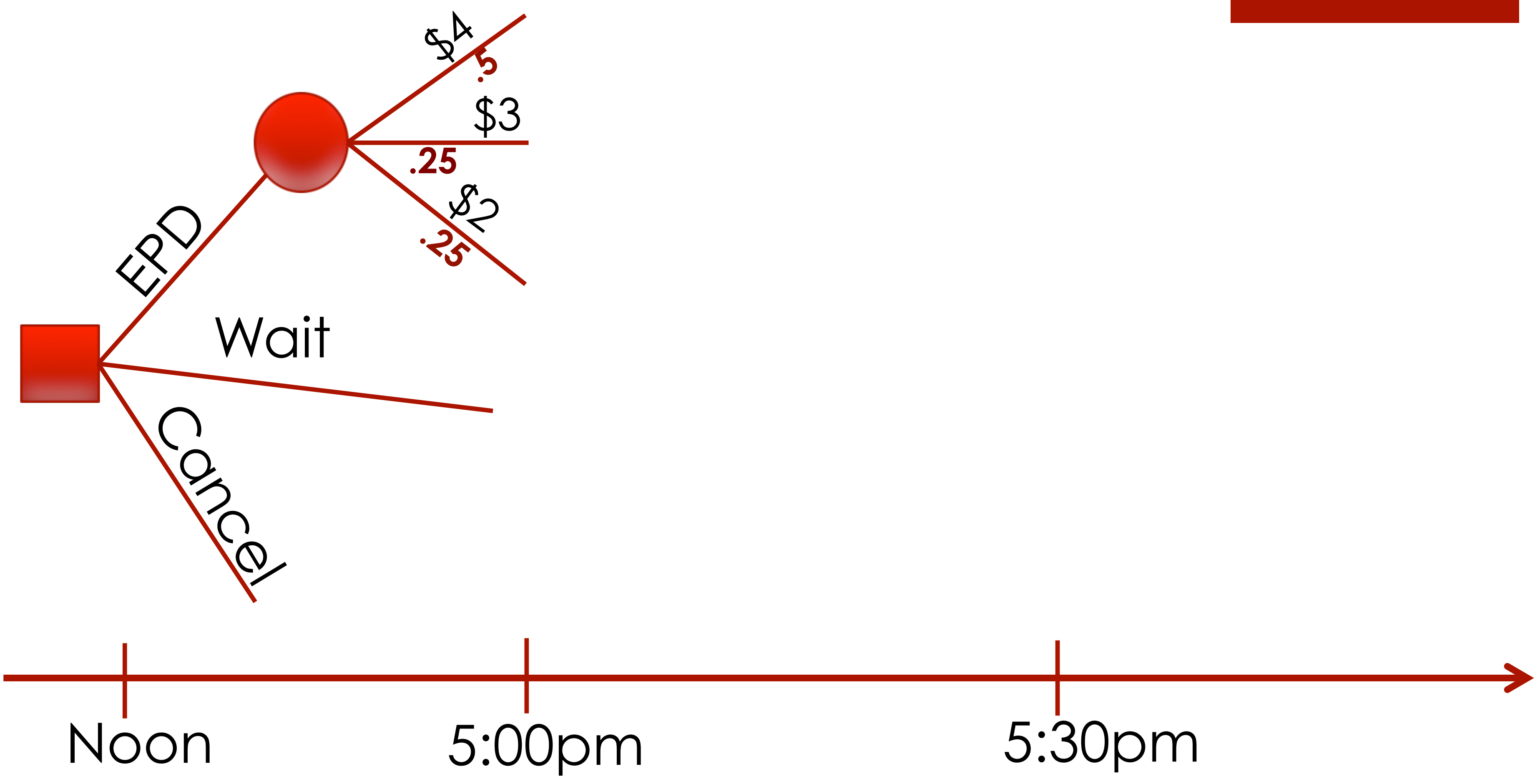
Logan Airport:
- Closed w.p. 0.2
- Open w.p. 0.8

5:30pm

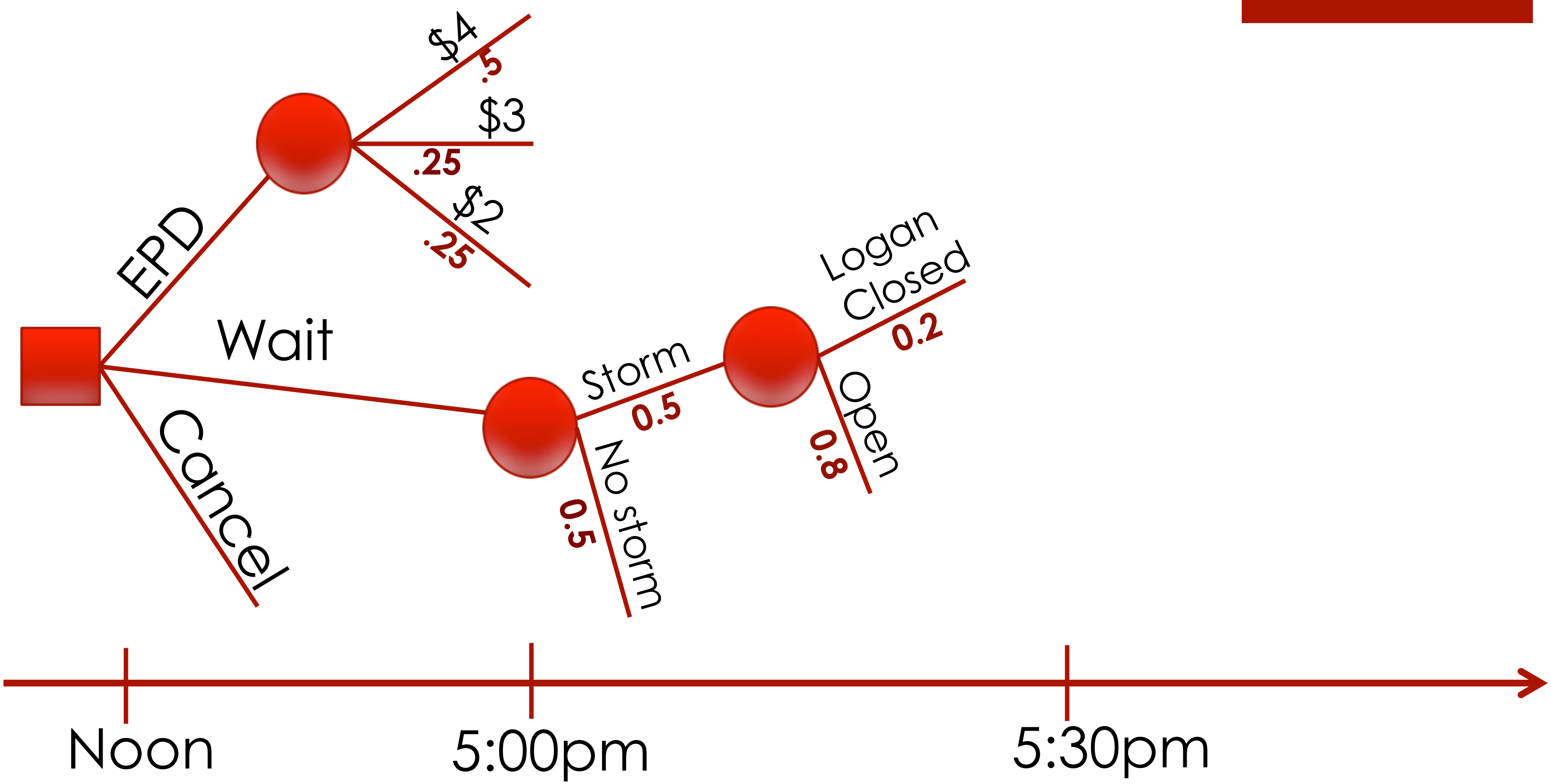
If MAF, cost is uncertain:
- $13 w.p. 0.67
- $19 w.p. 0.33

If order canceled:
- Redeem w.p. 0.7
- Not w.p. 0.3

Decision Tree



Decision Tree



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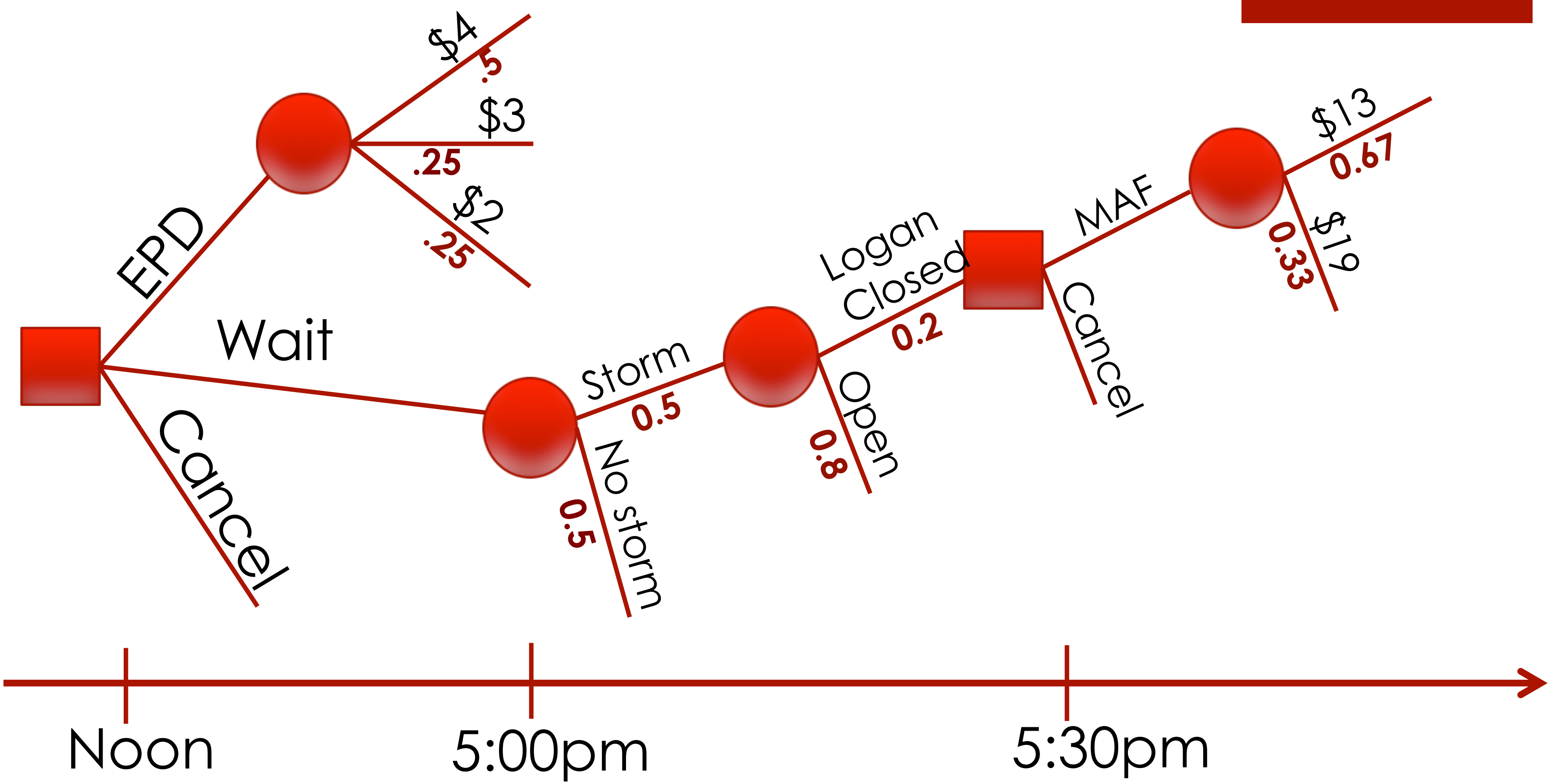
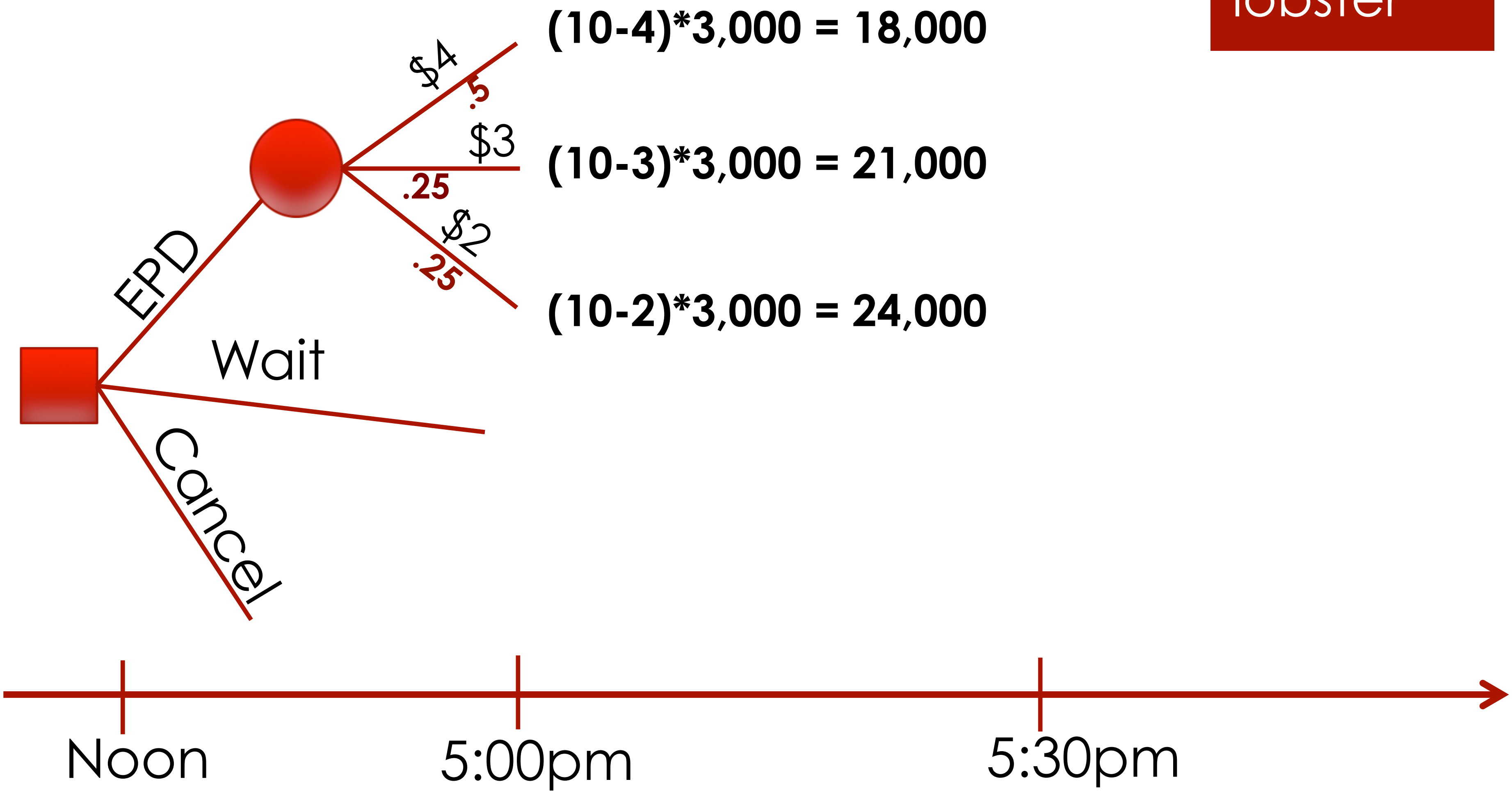


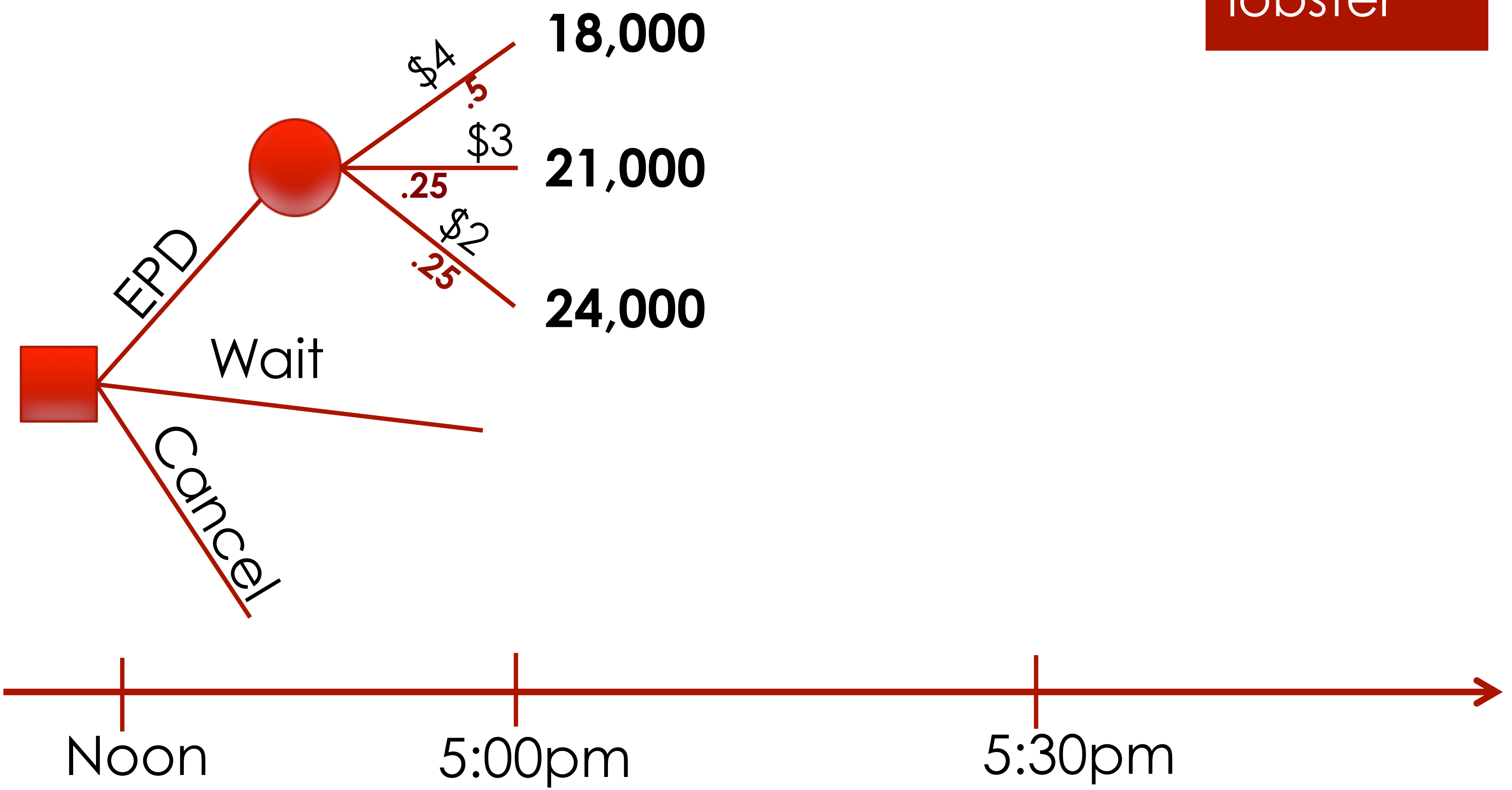
Calculating the Branch Values

$10 profit/ lobster



Calculating the Branch Values

$10 profit/ lobster



Calculating the Branch Values

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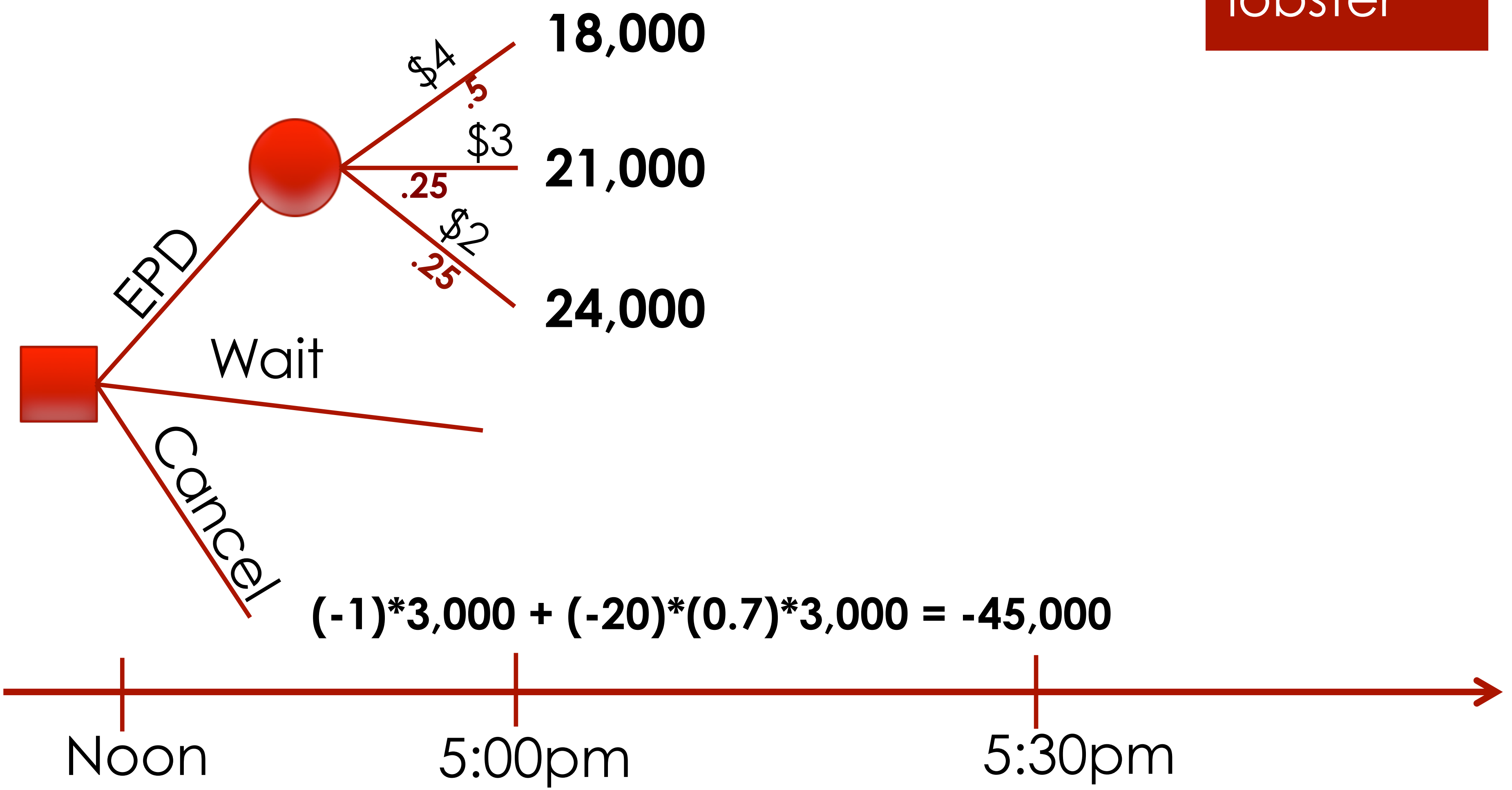

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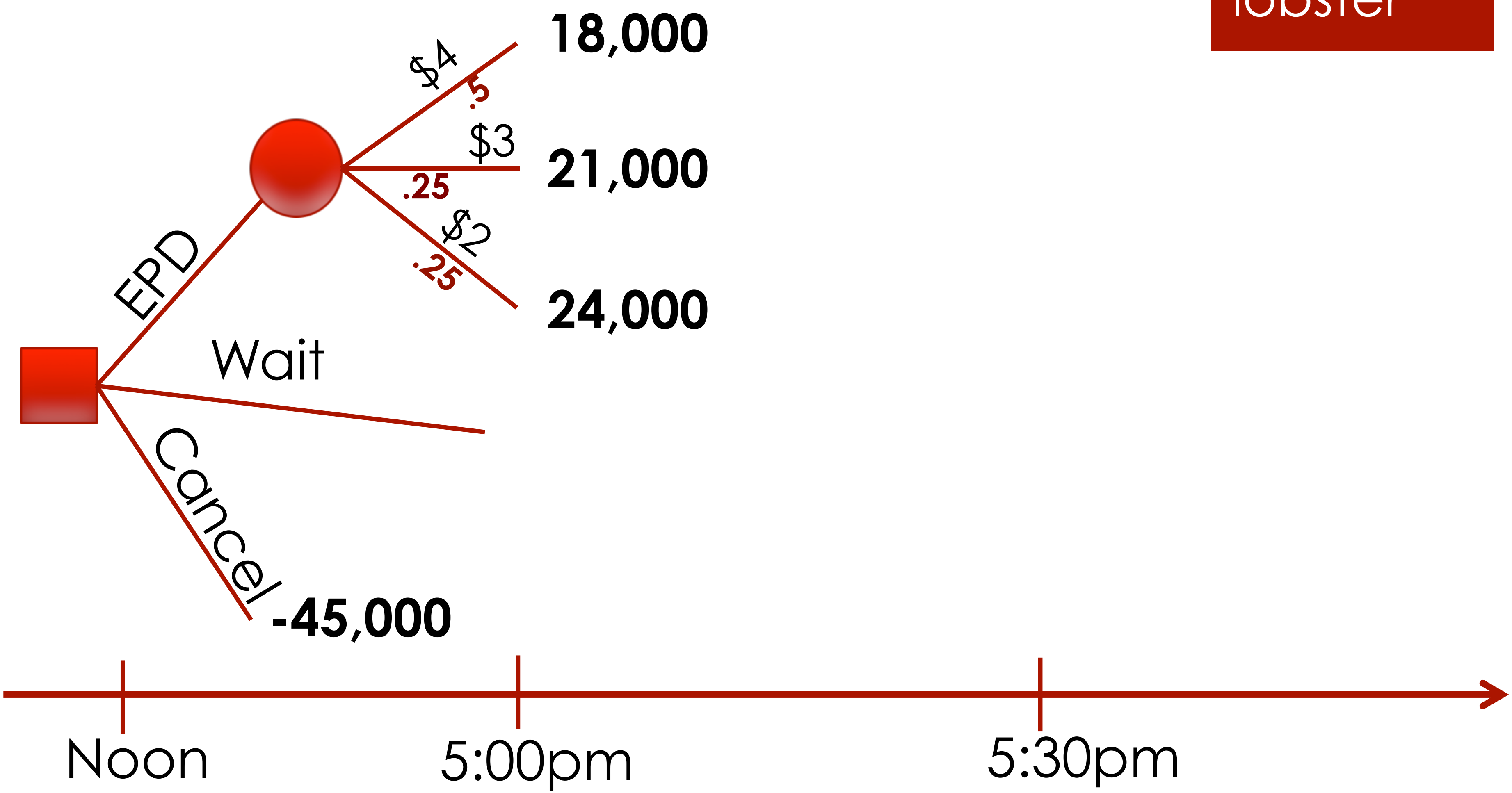
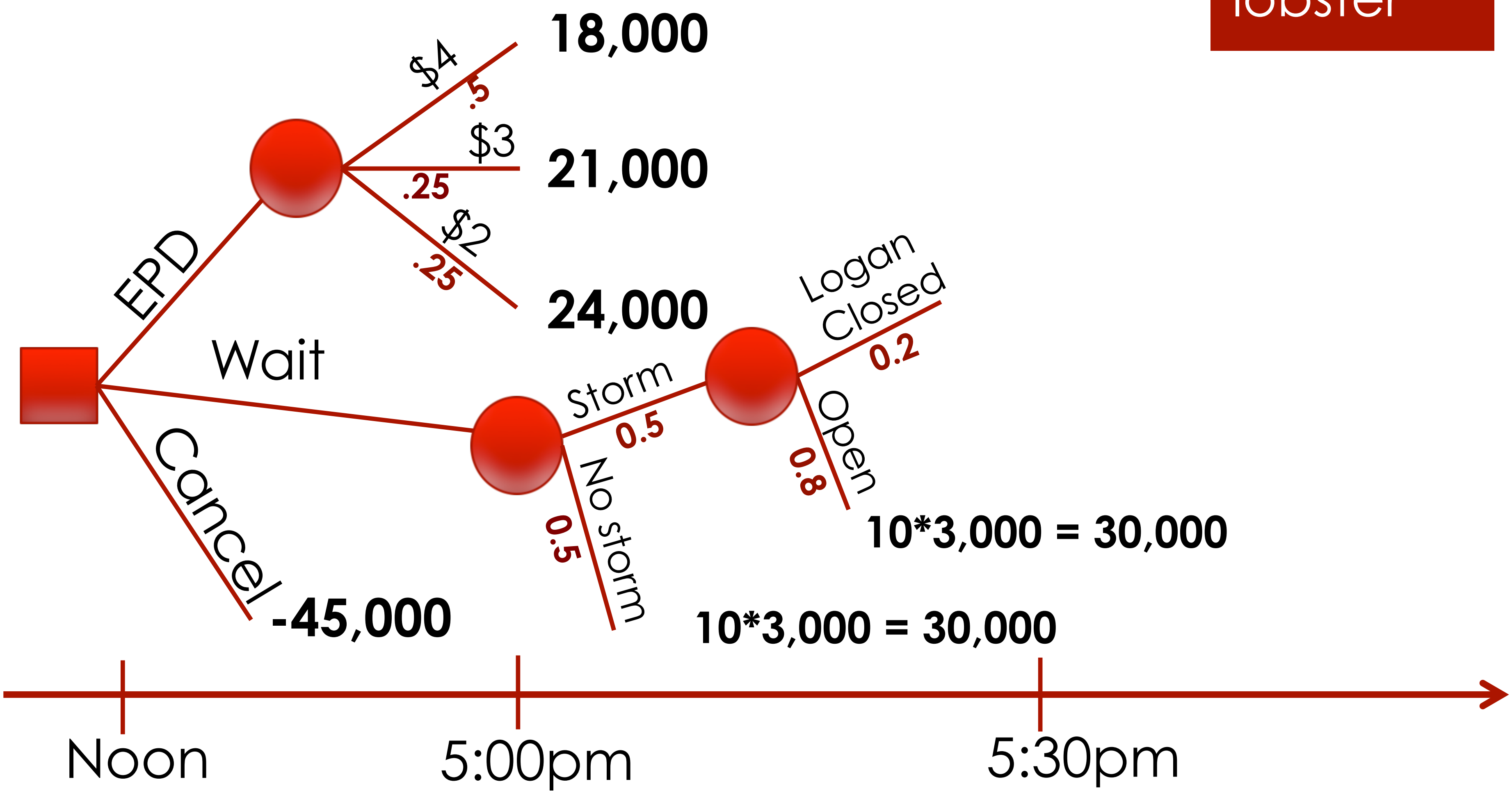


Calculating the Branch Values

32

$10 profit/ lobster



Calculating the Branch Values

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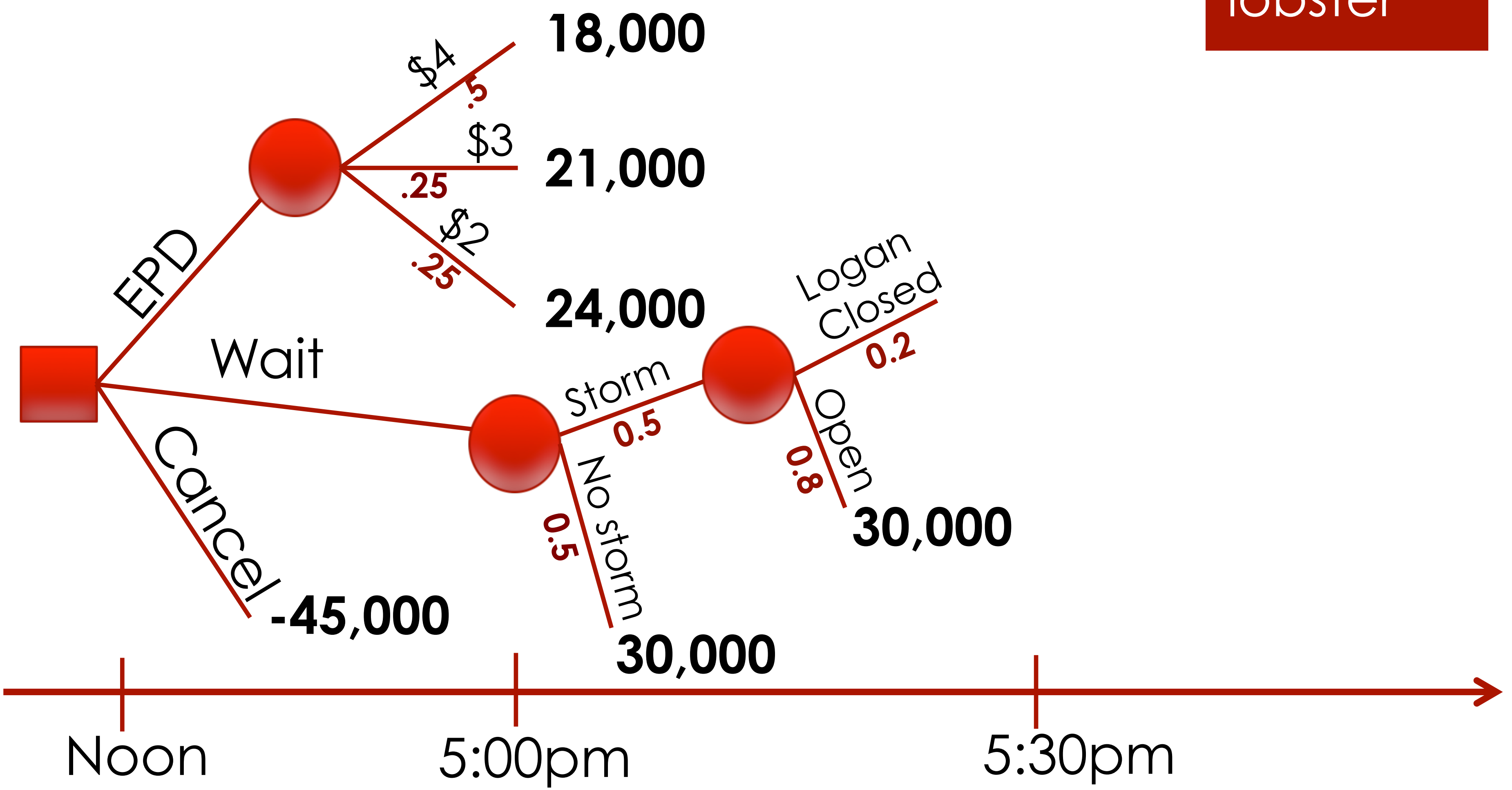

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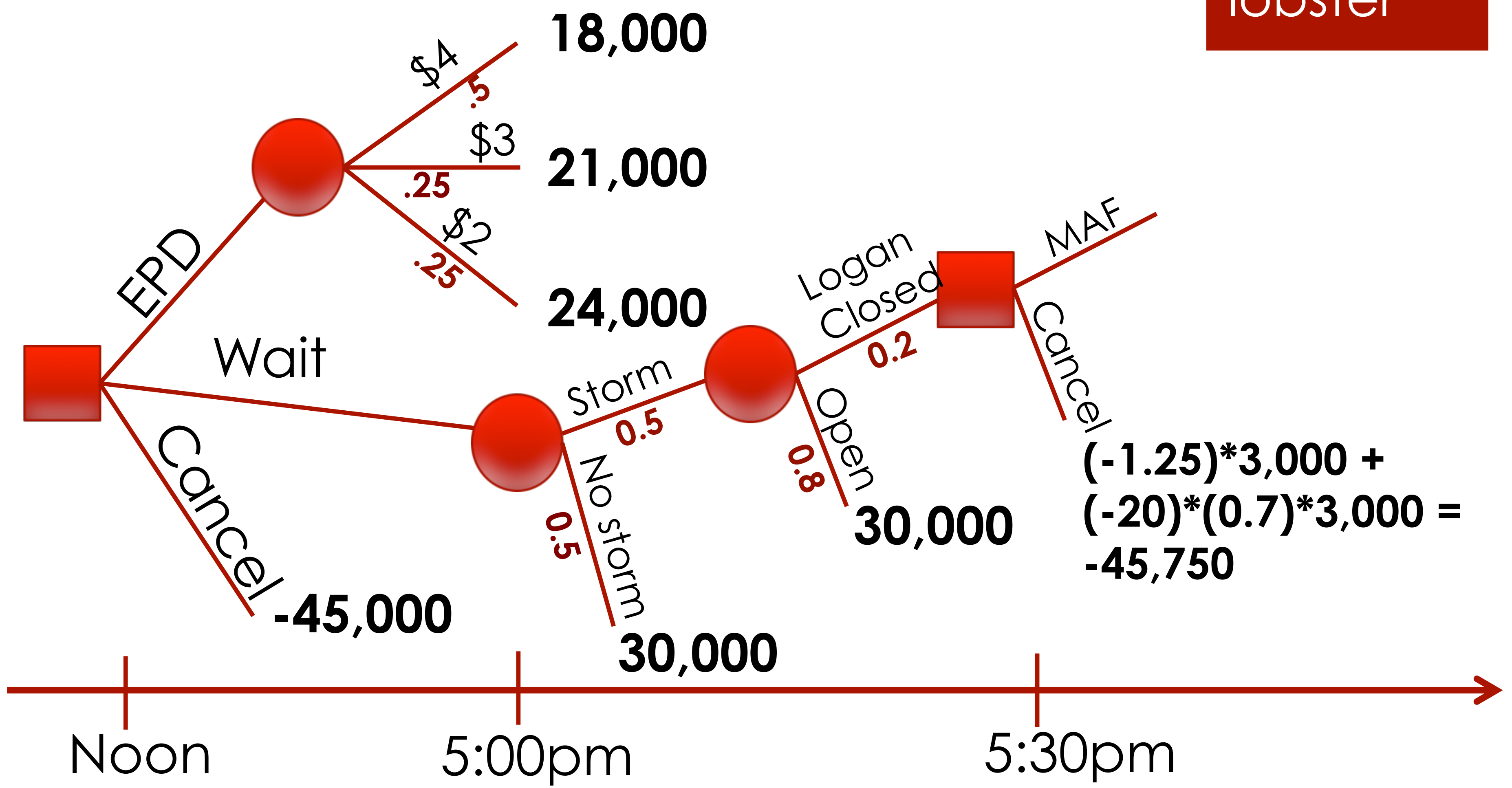

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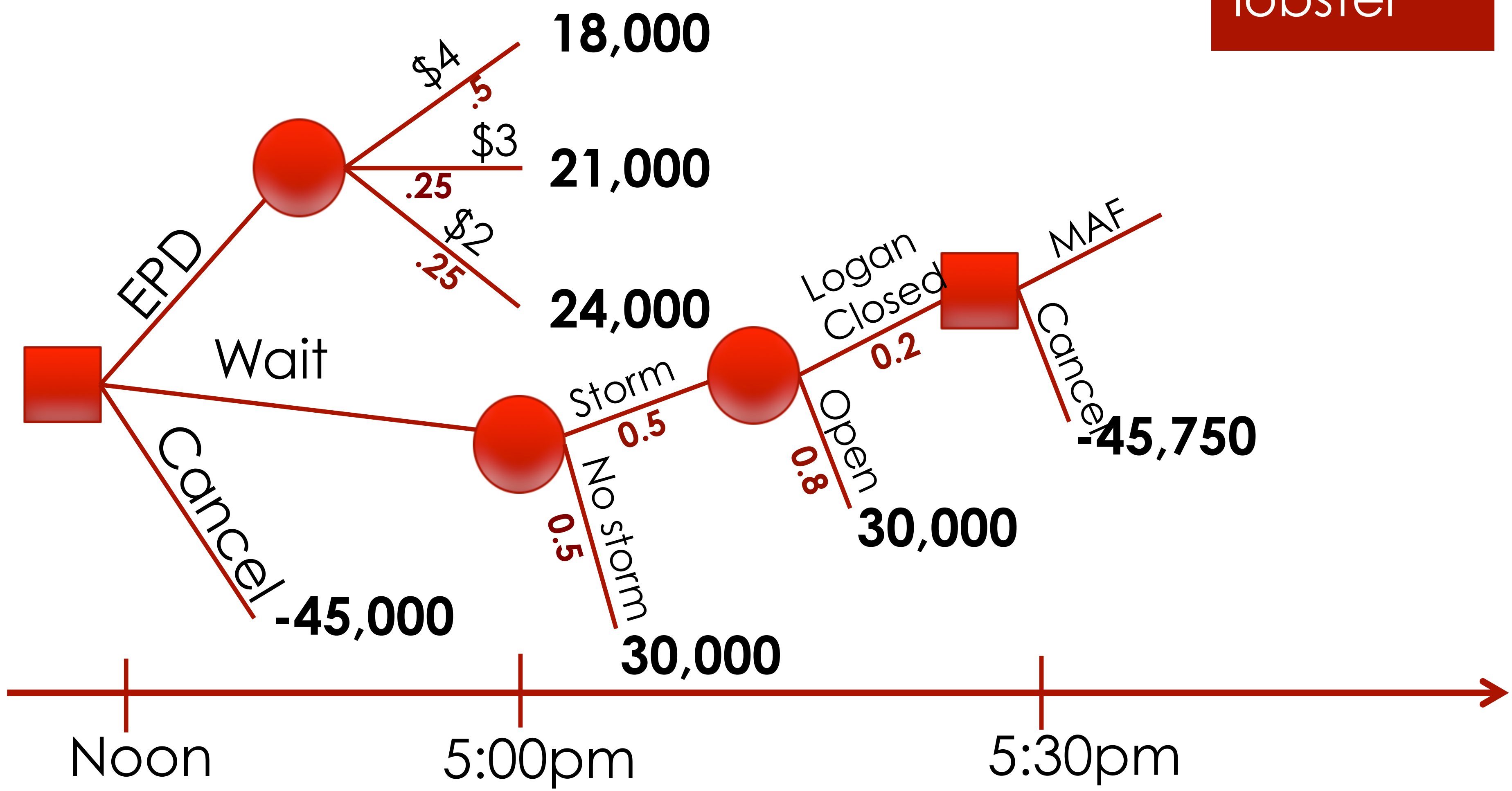

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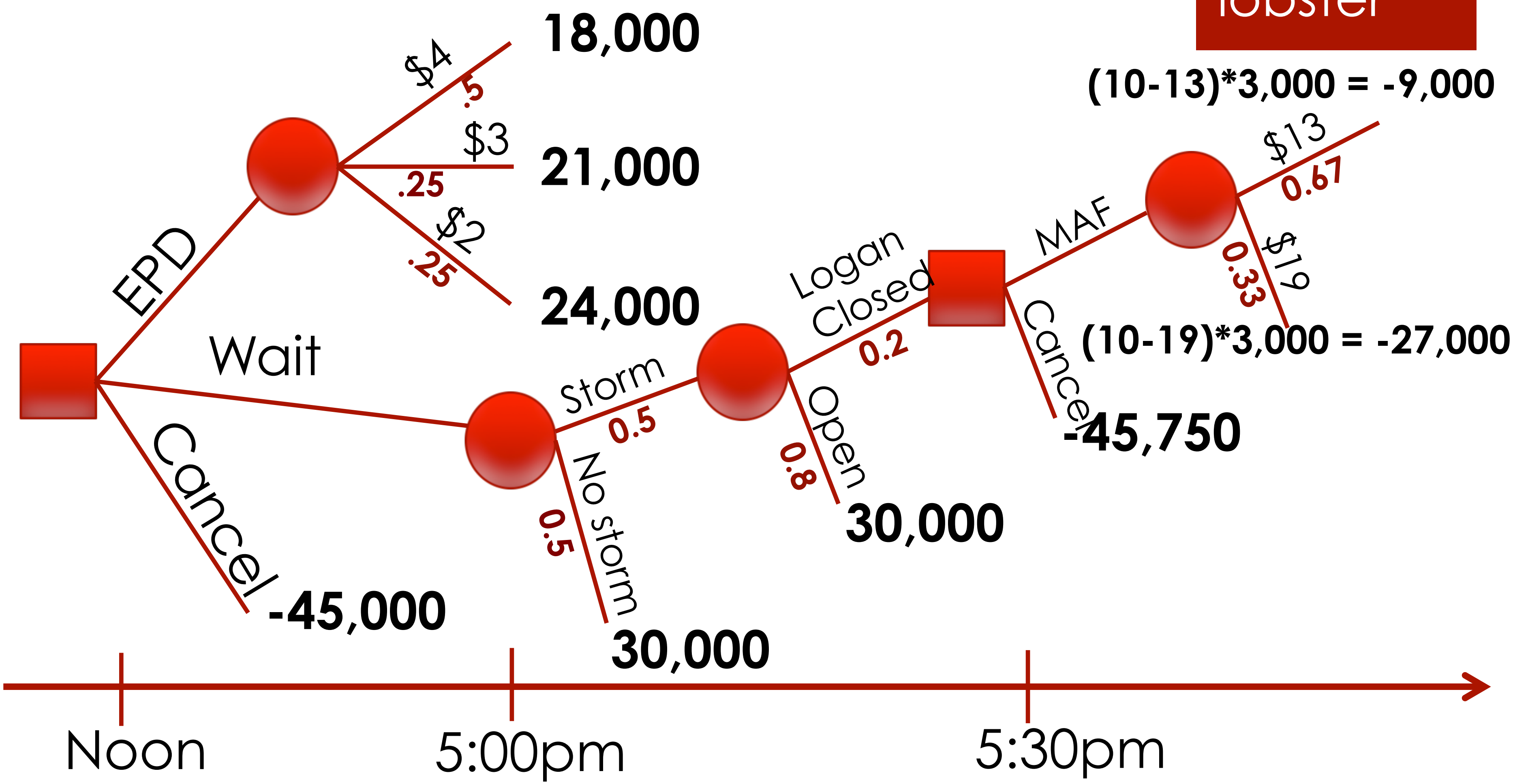

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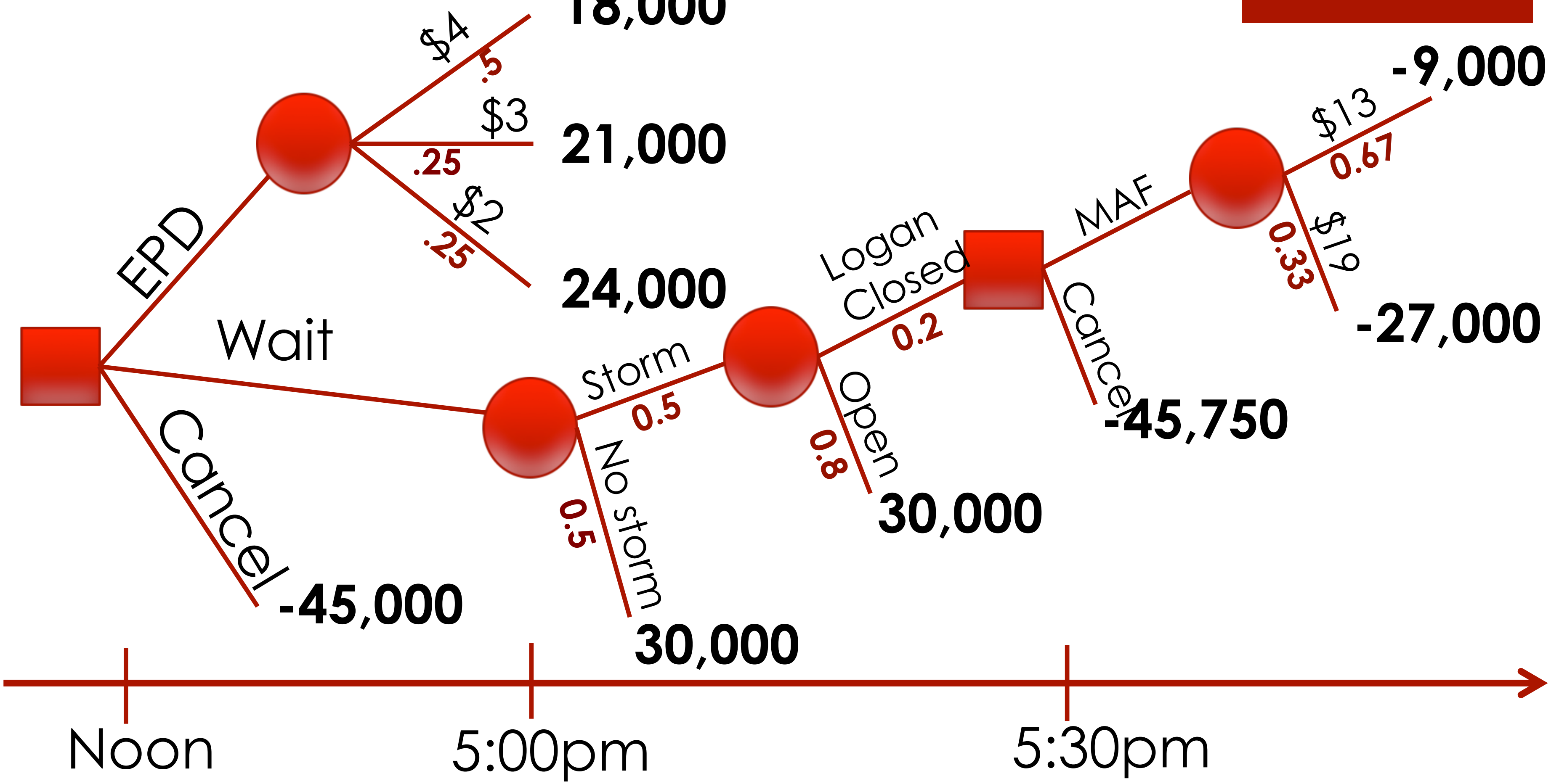
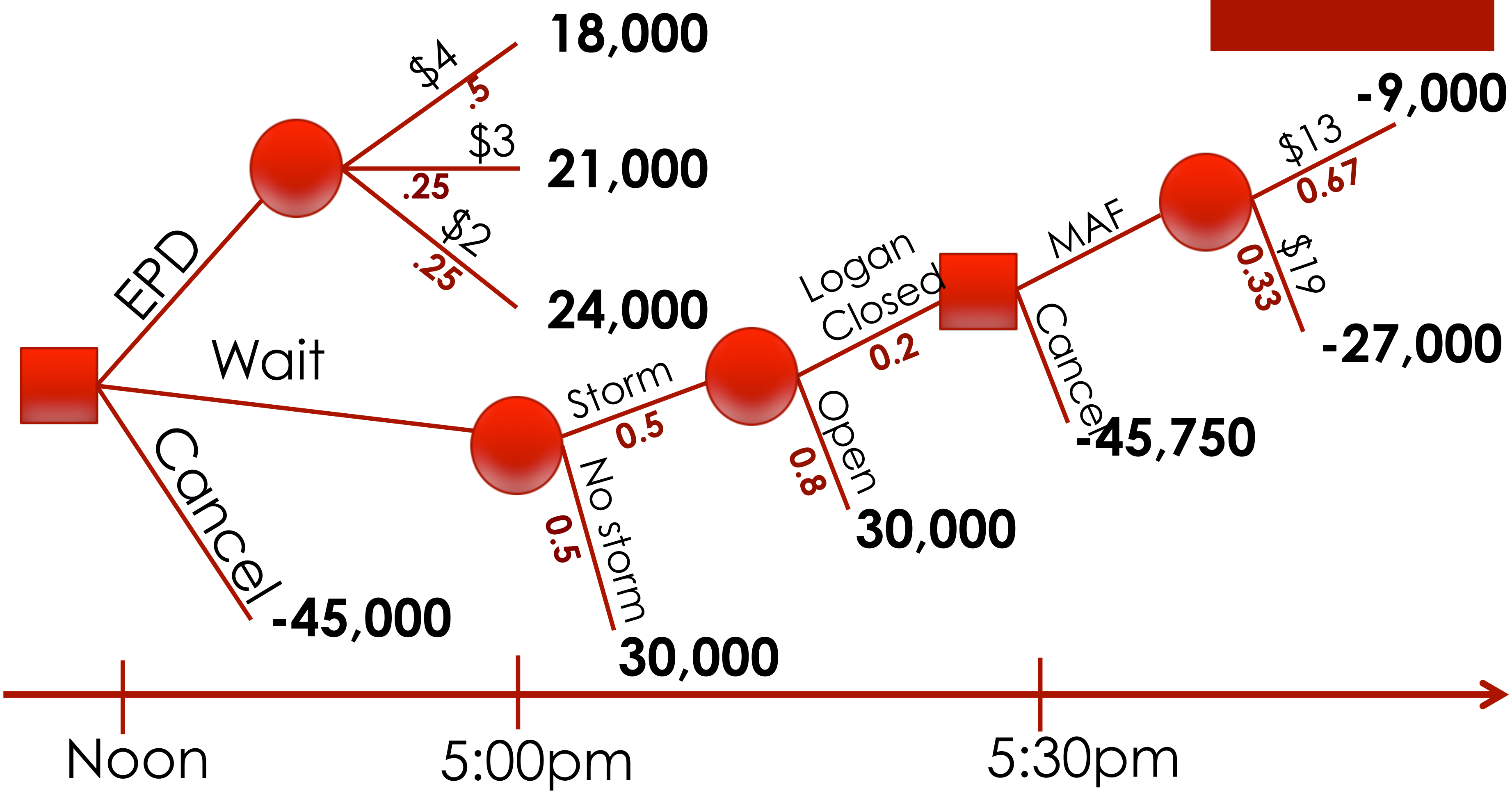


Calculating the EMV



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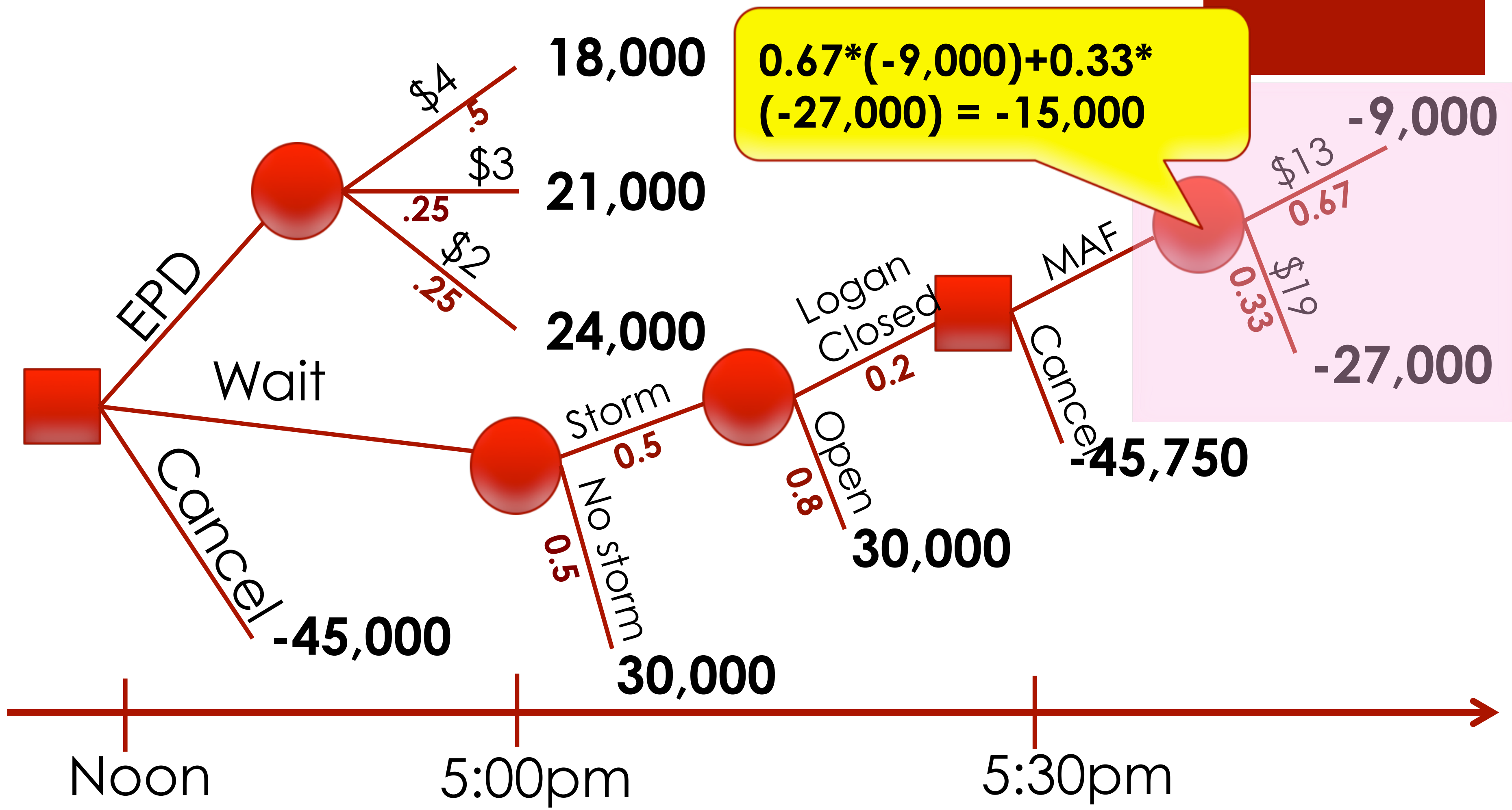

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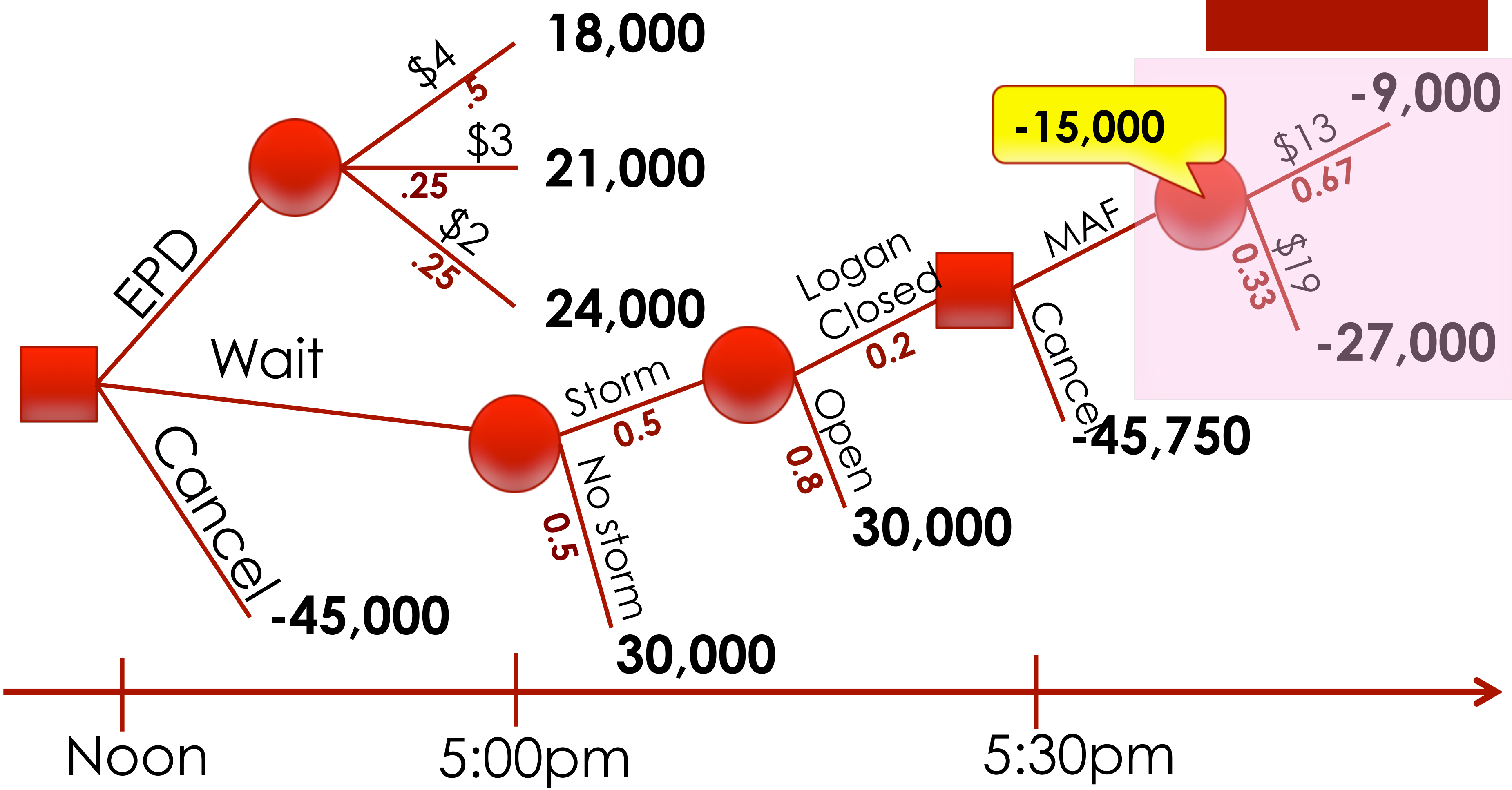
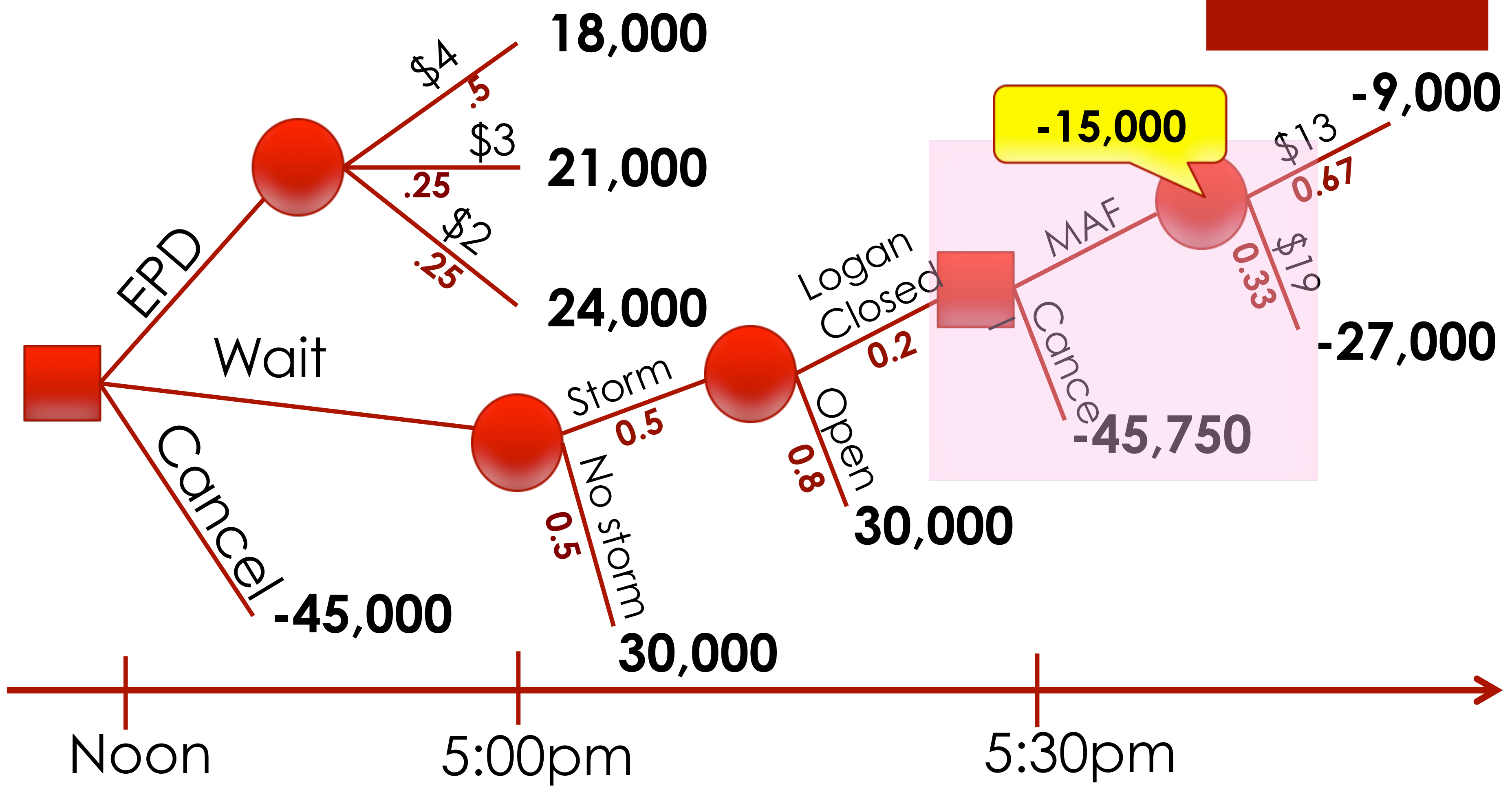


Comparing the EMV



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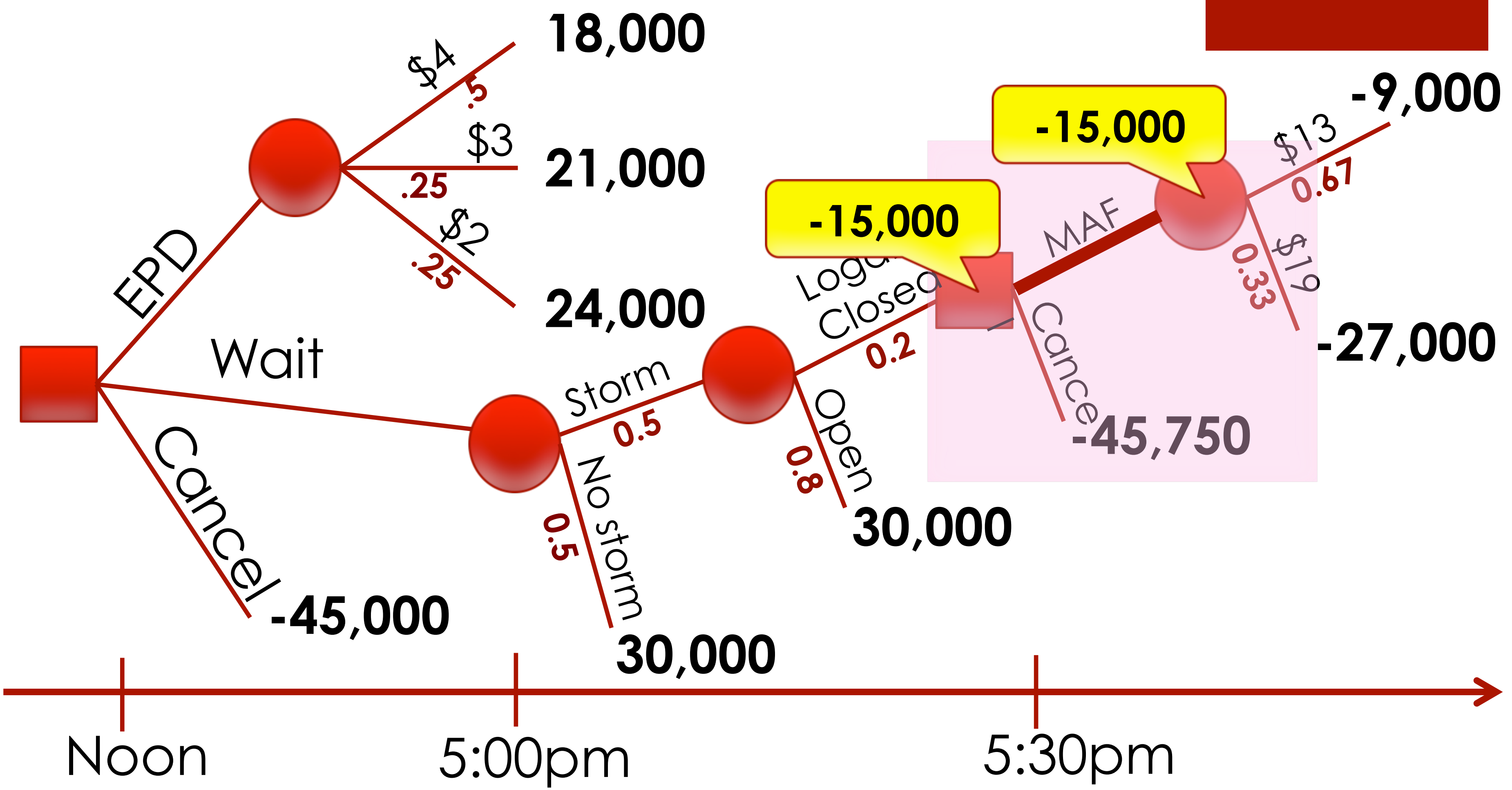

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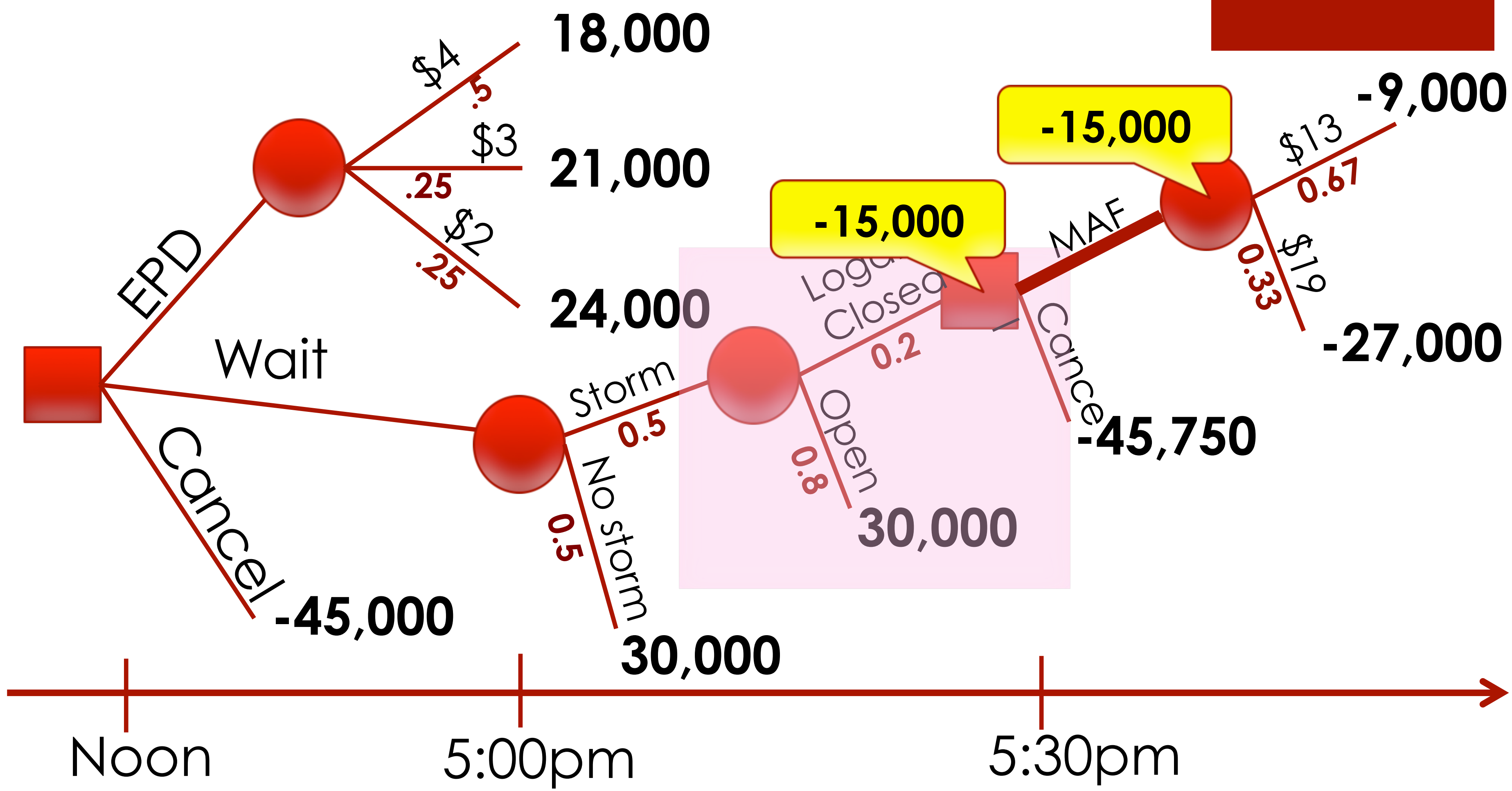

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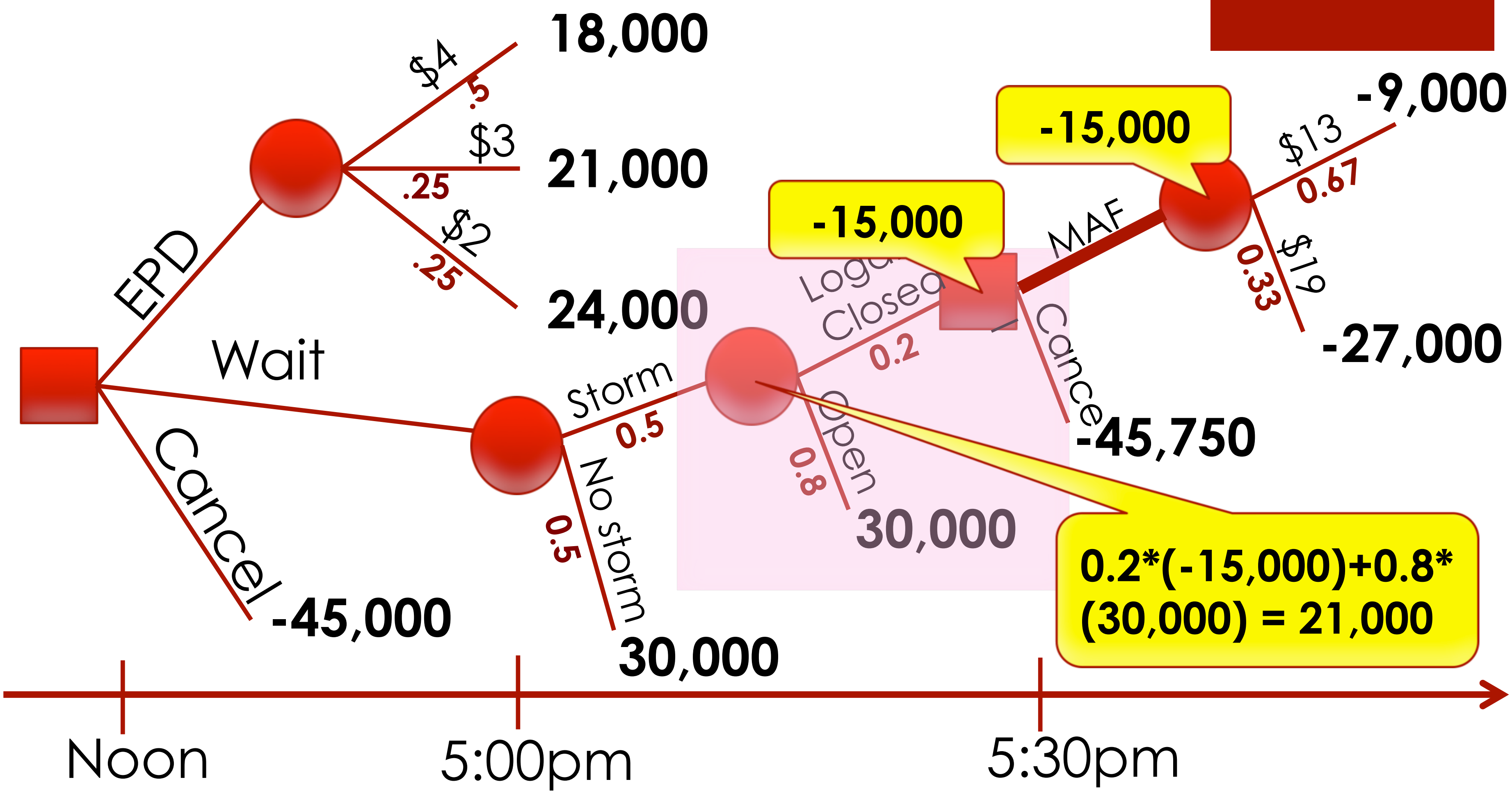

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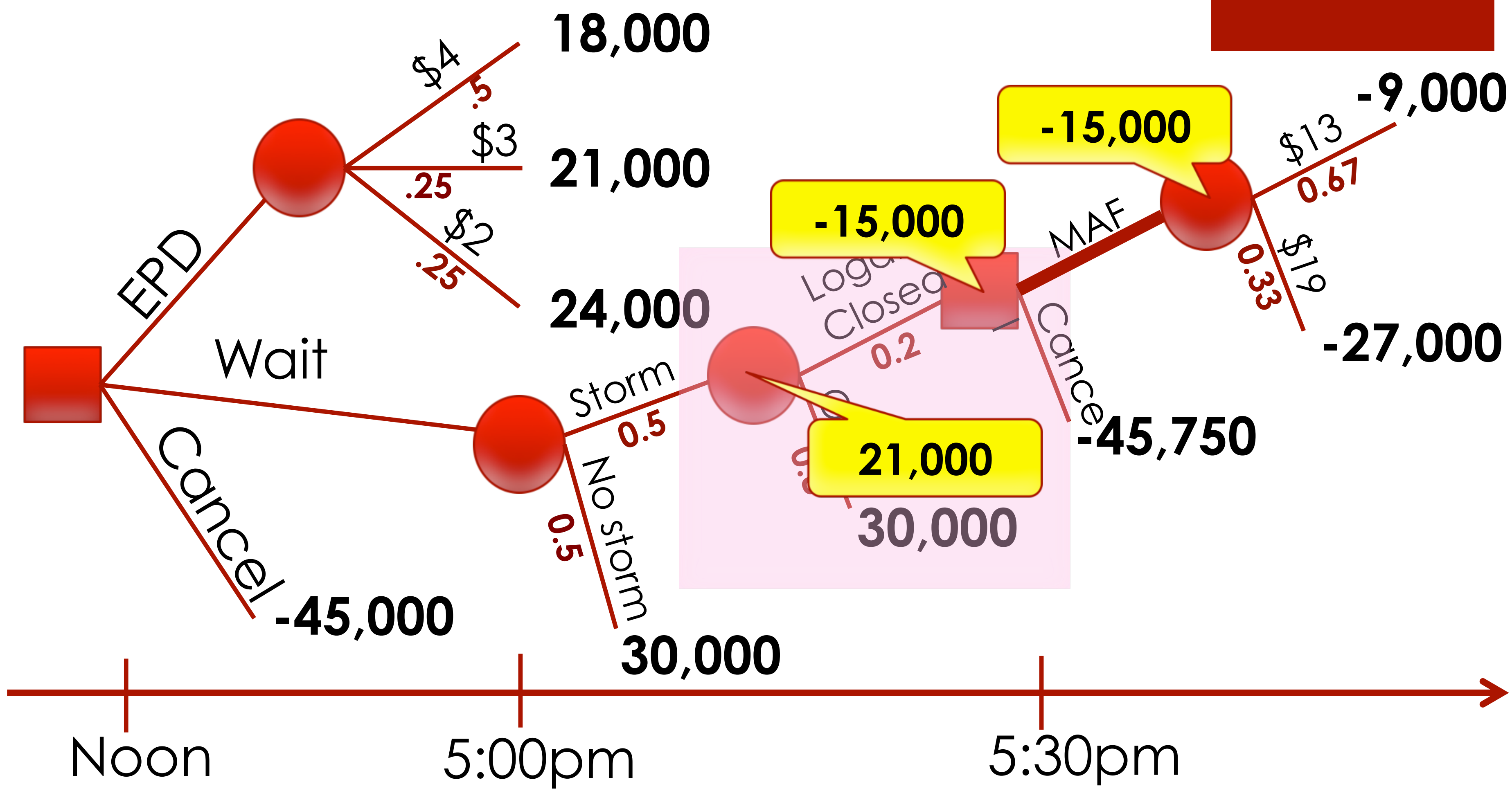

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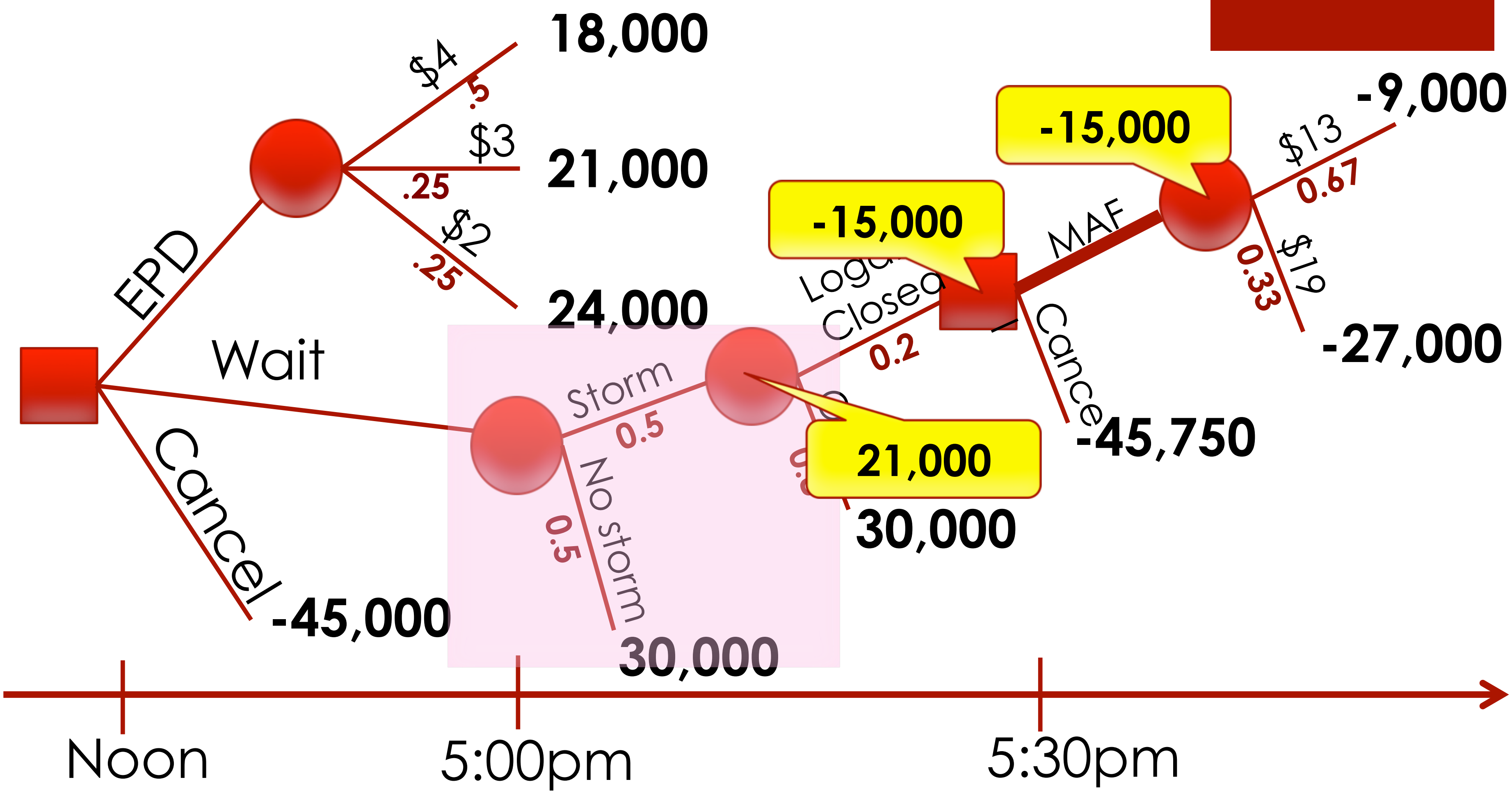

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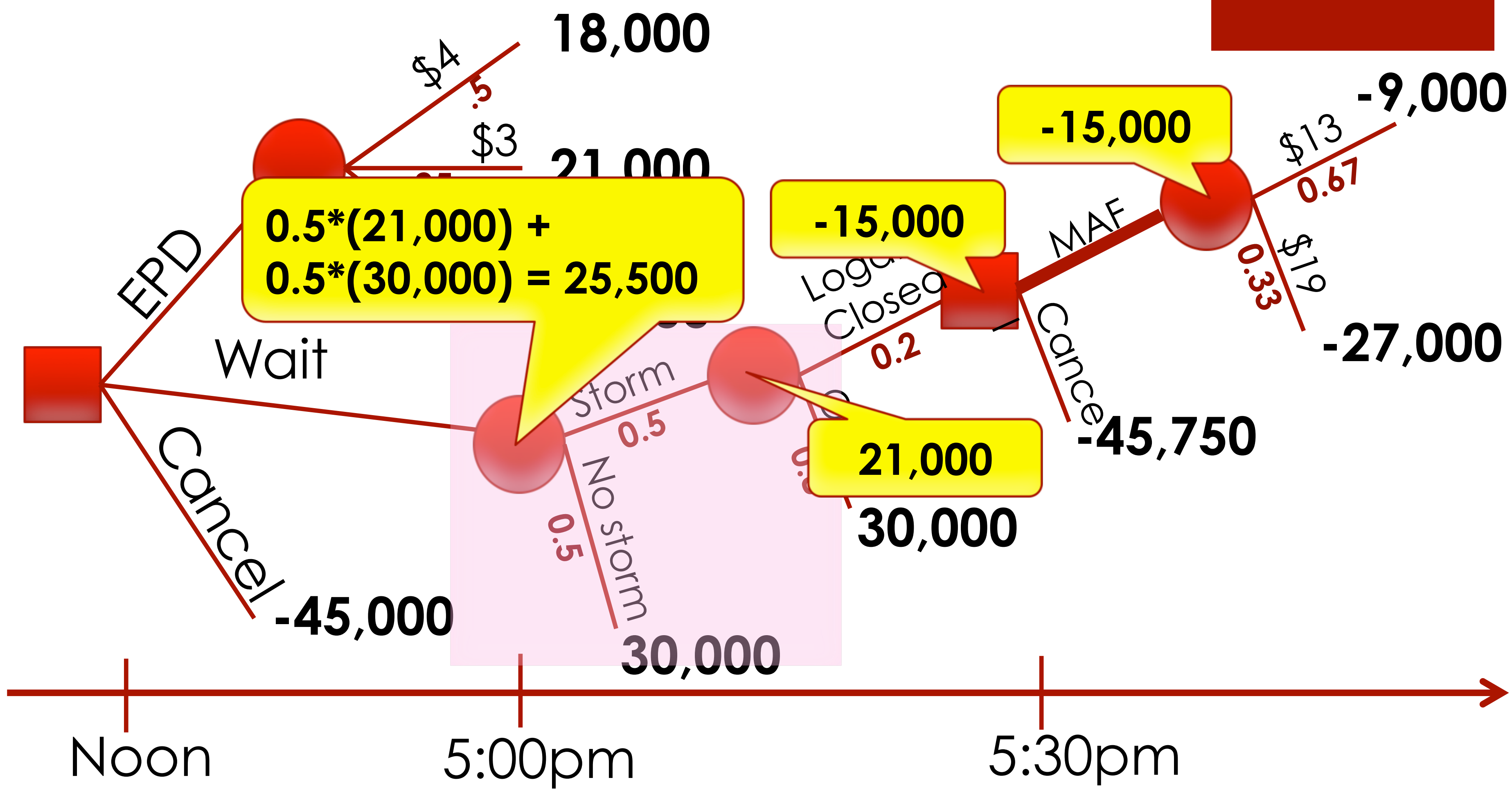

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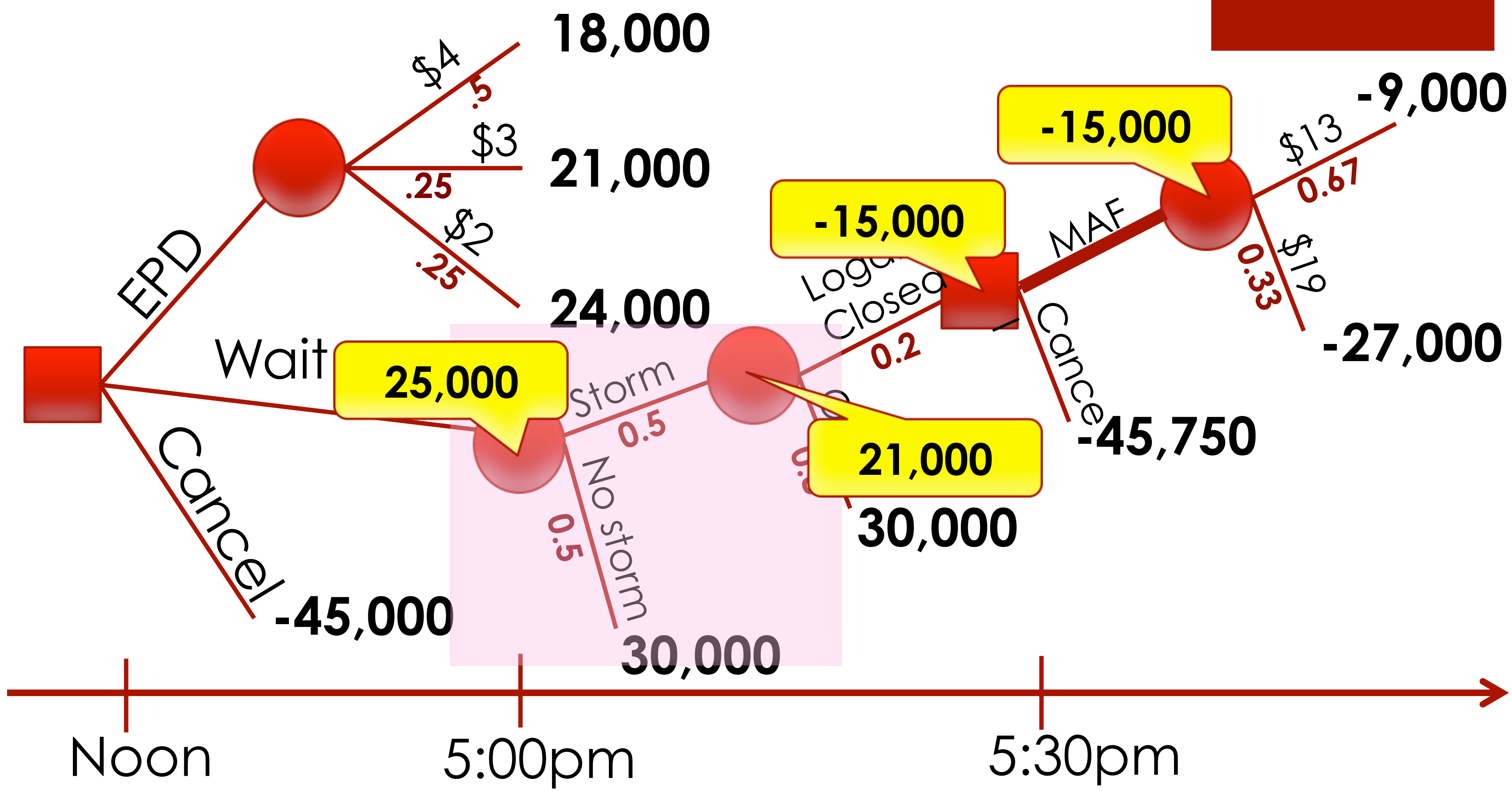

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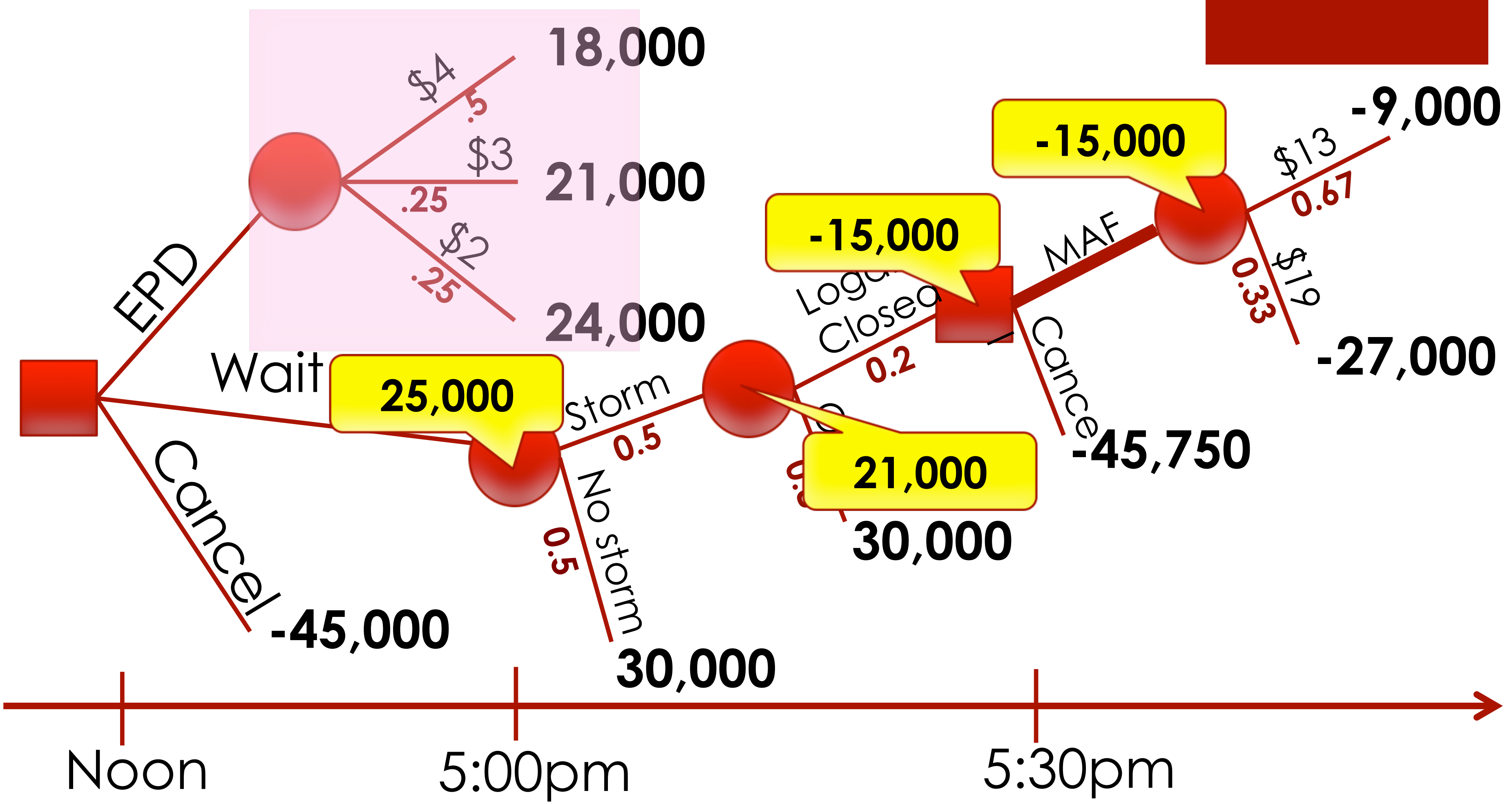

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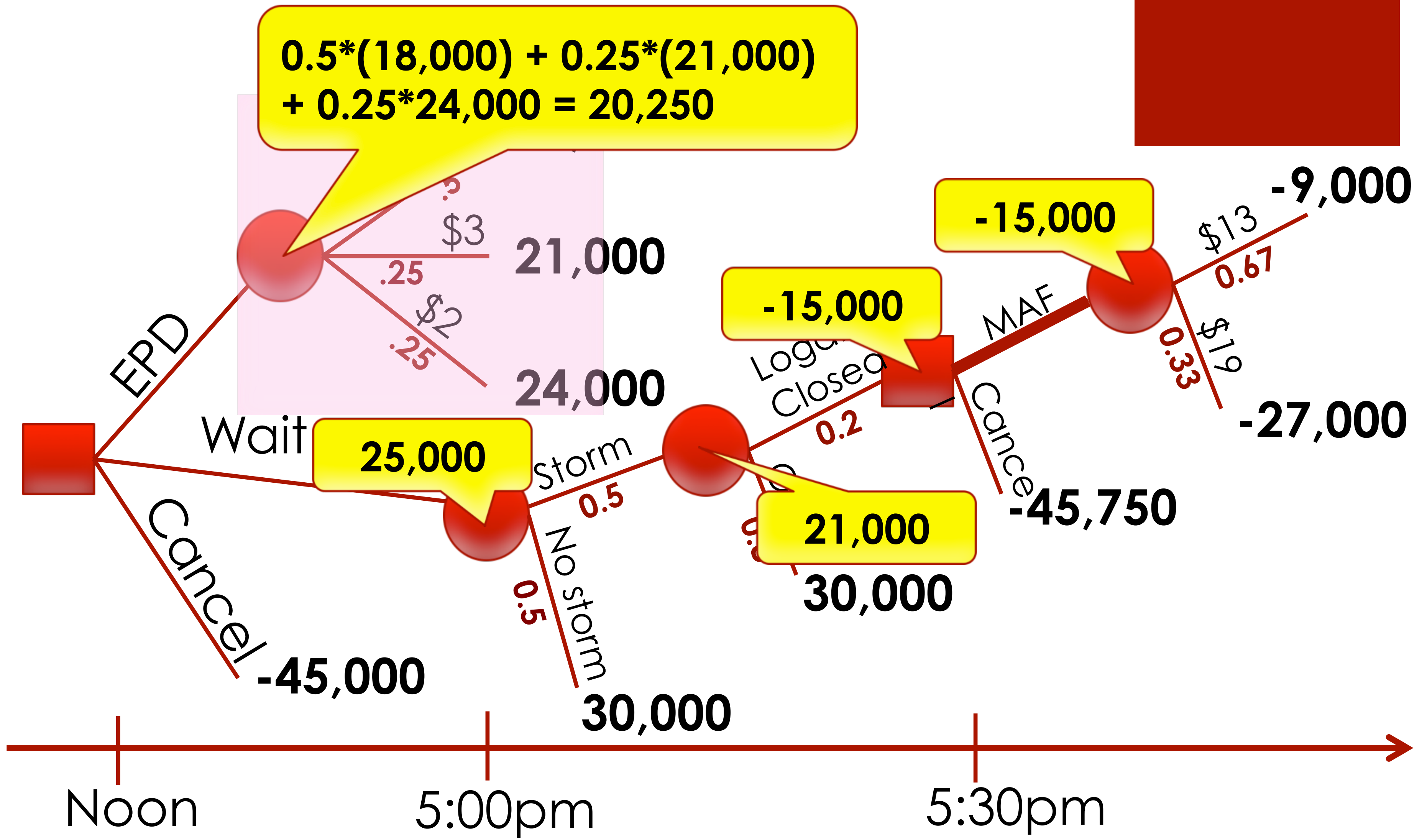

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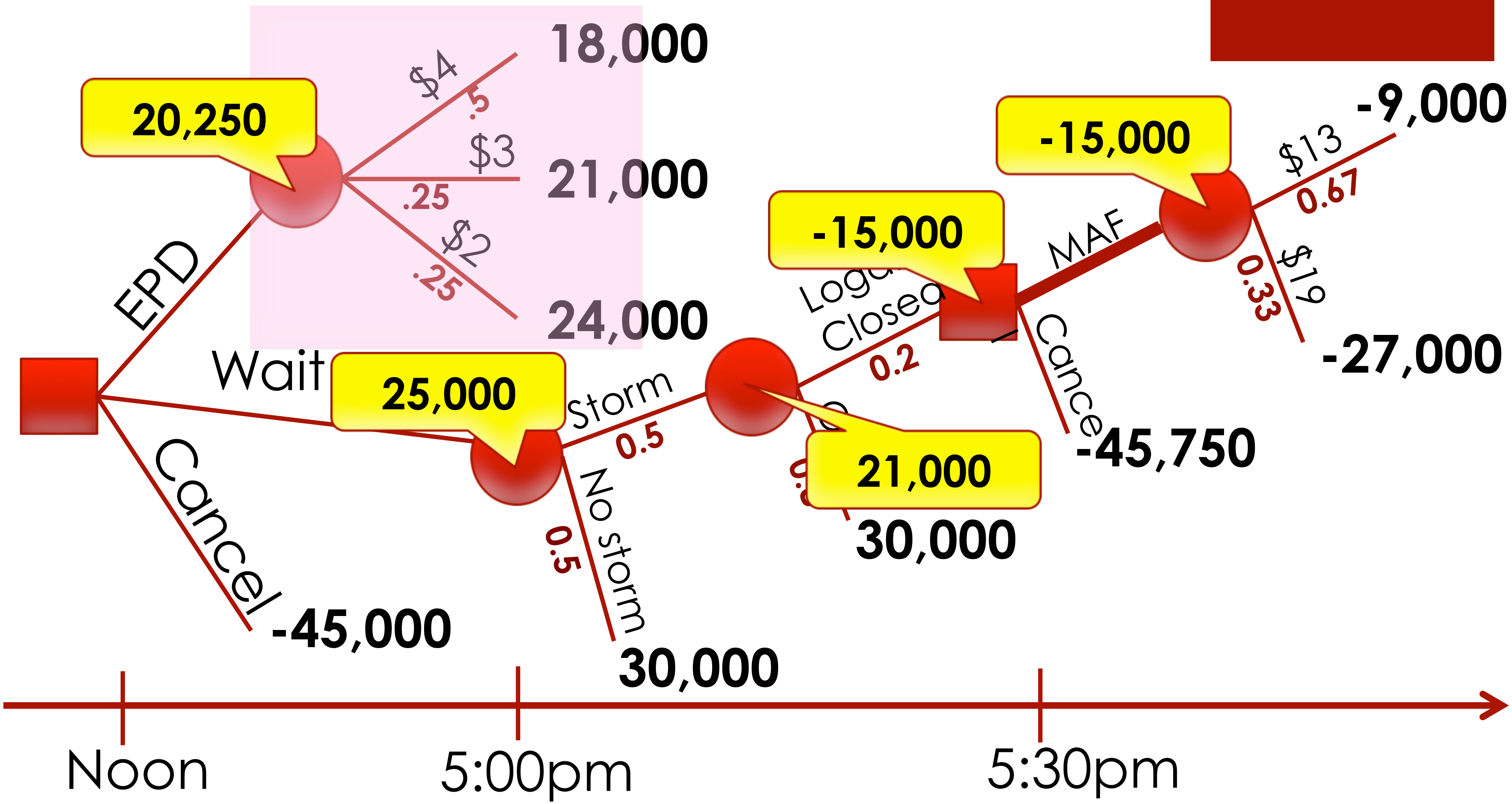

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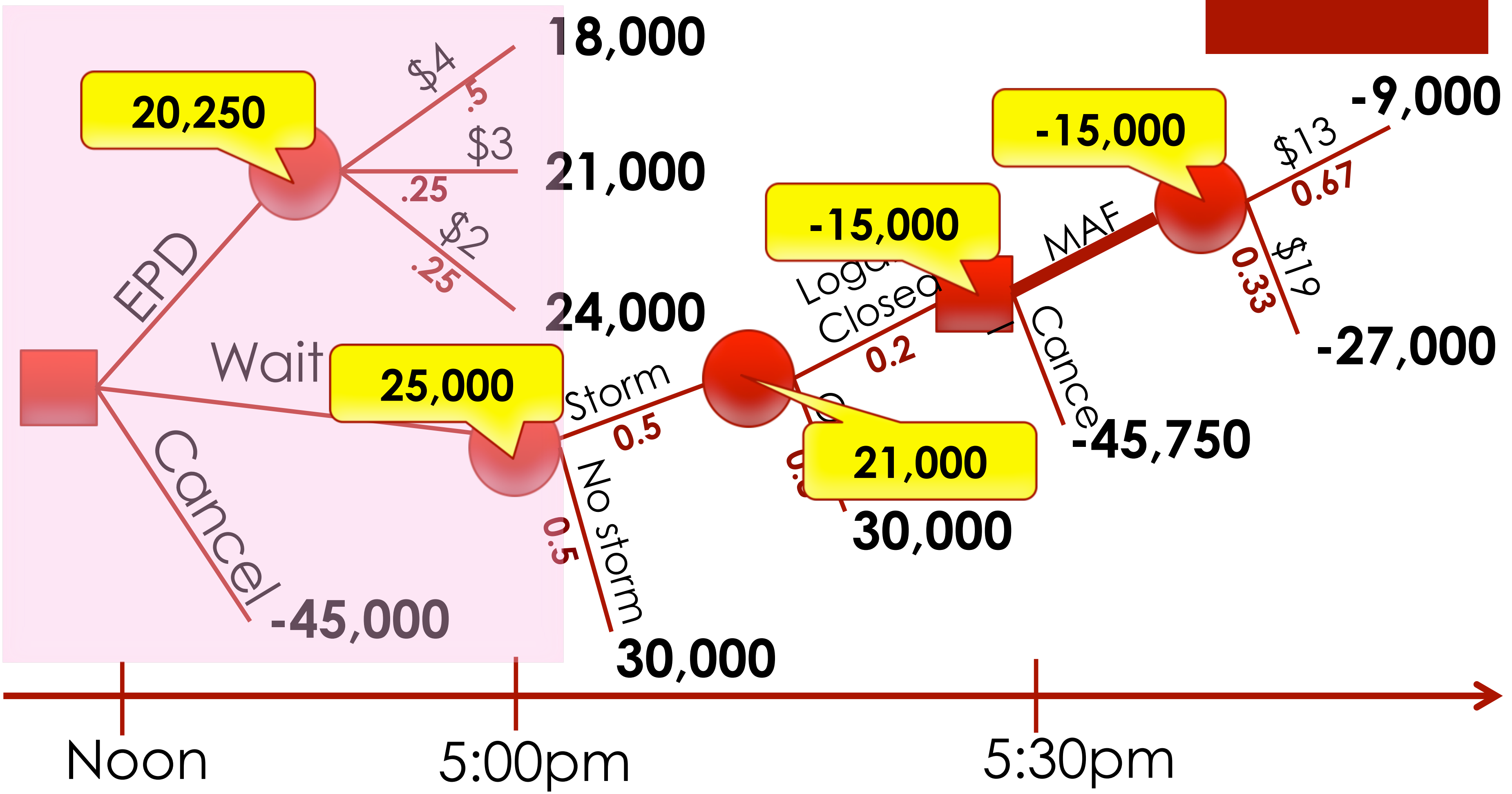

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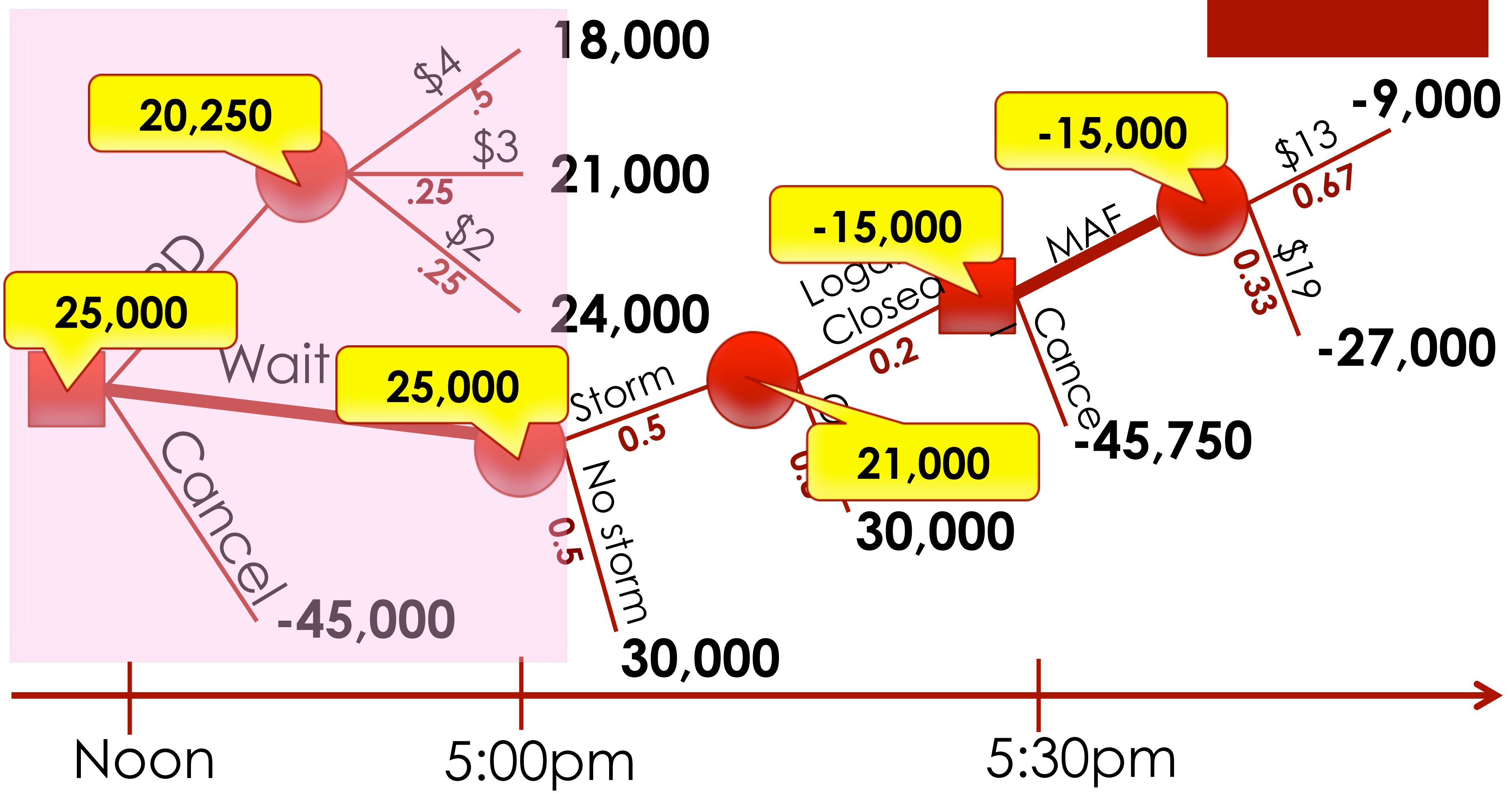
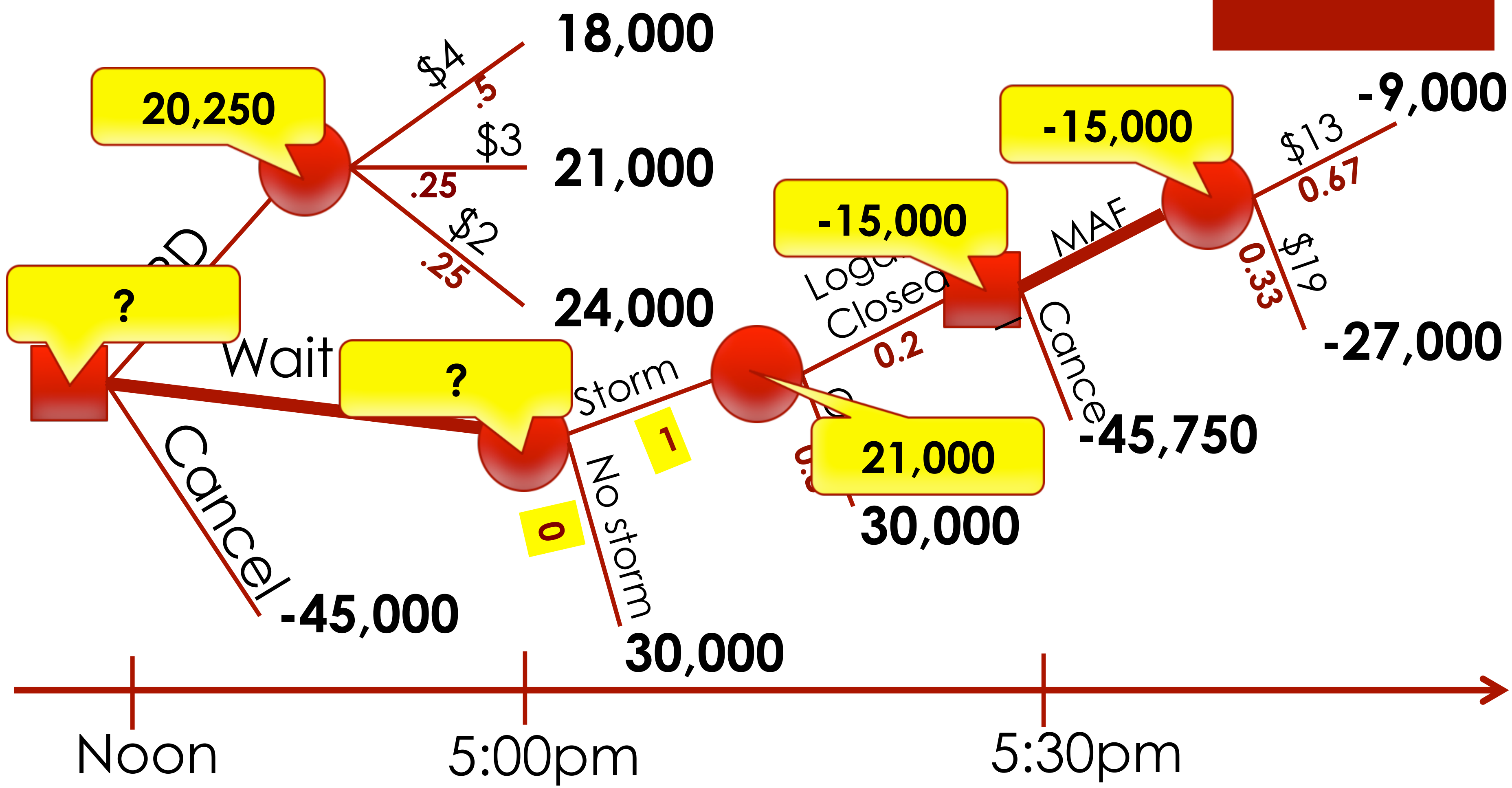


Sensitivity Analysis



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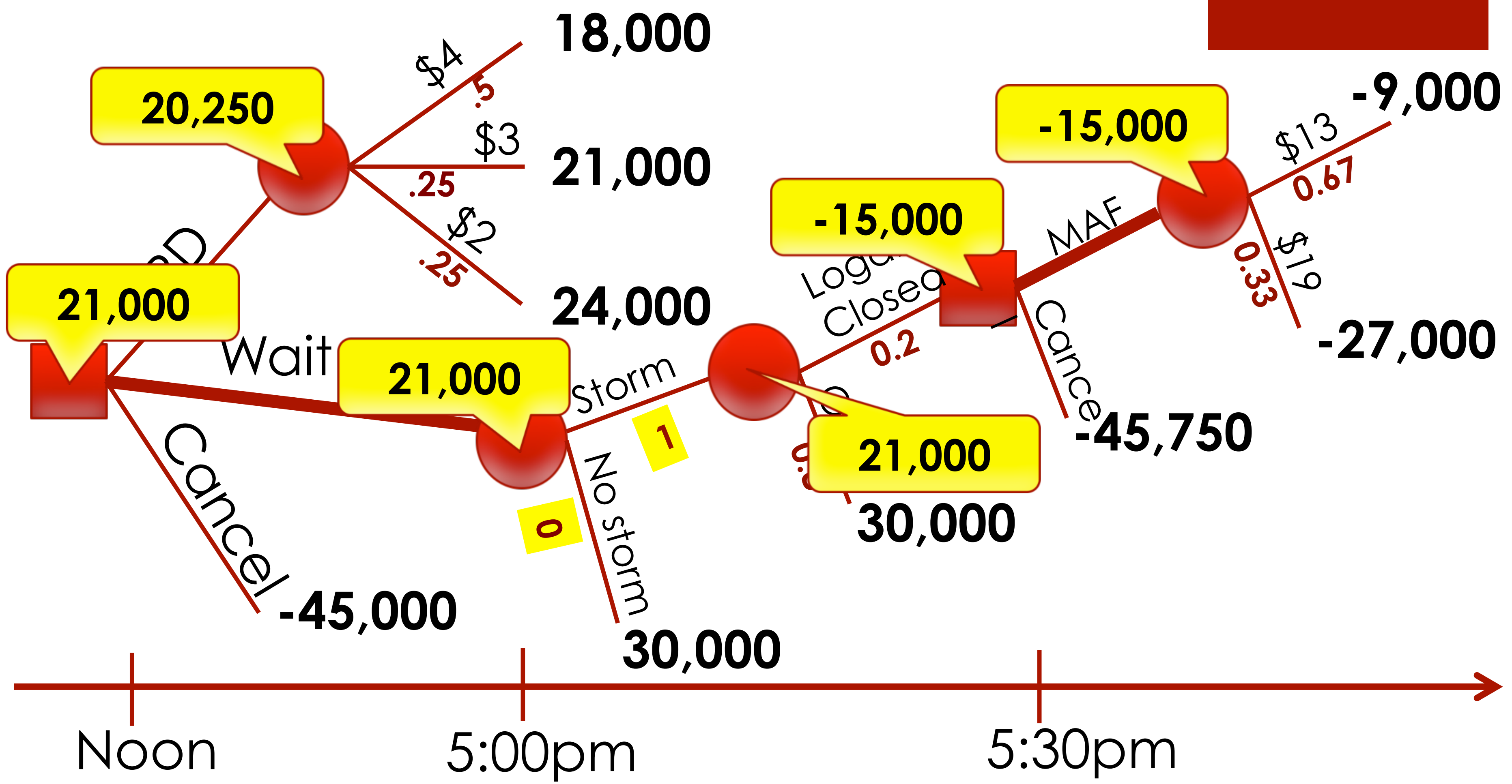
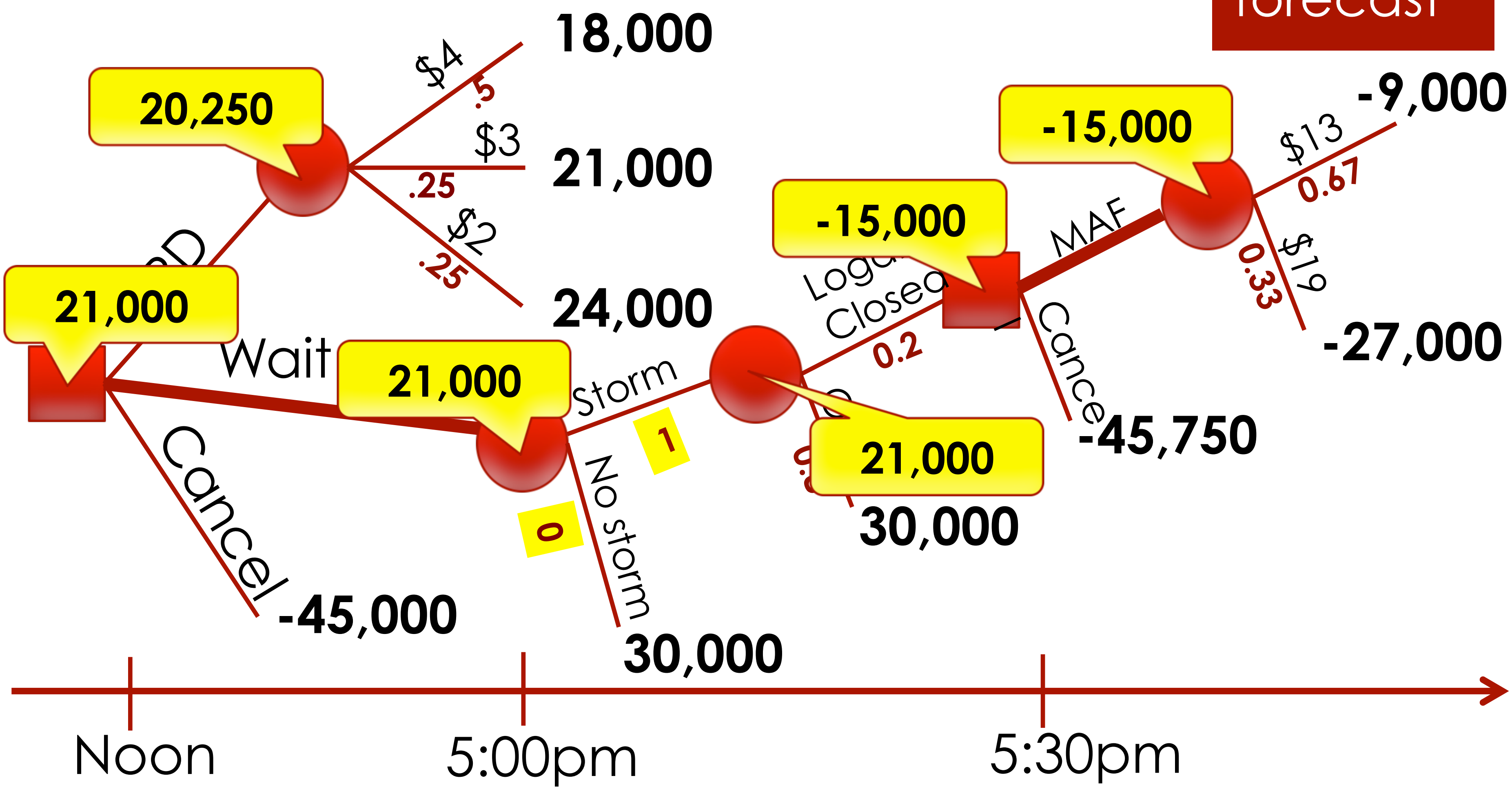


Sensitivity Analysis

56
To wait is robust to forecast



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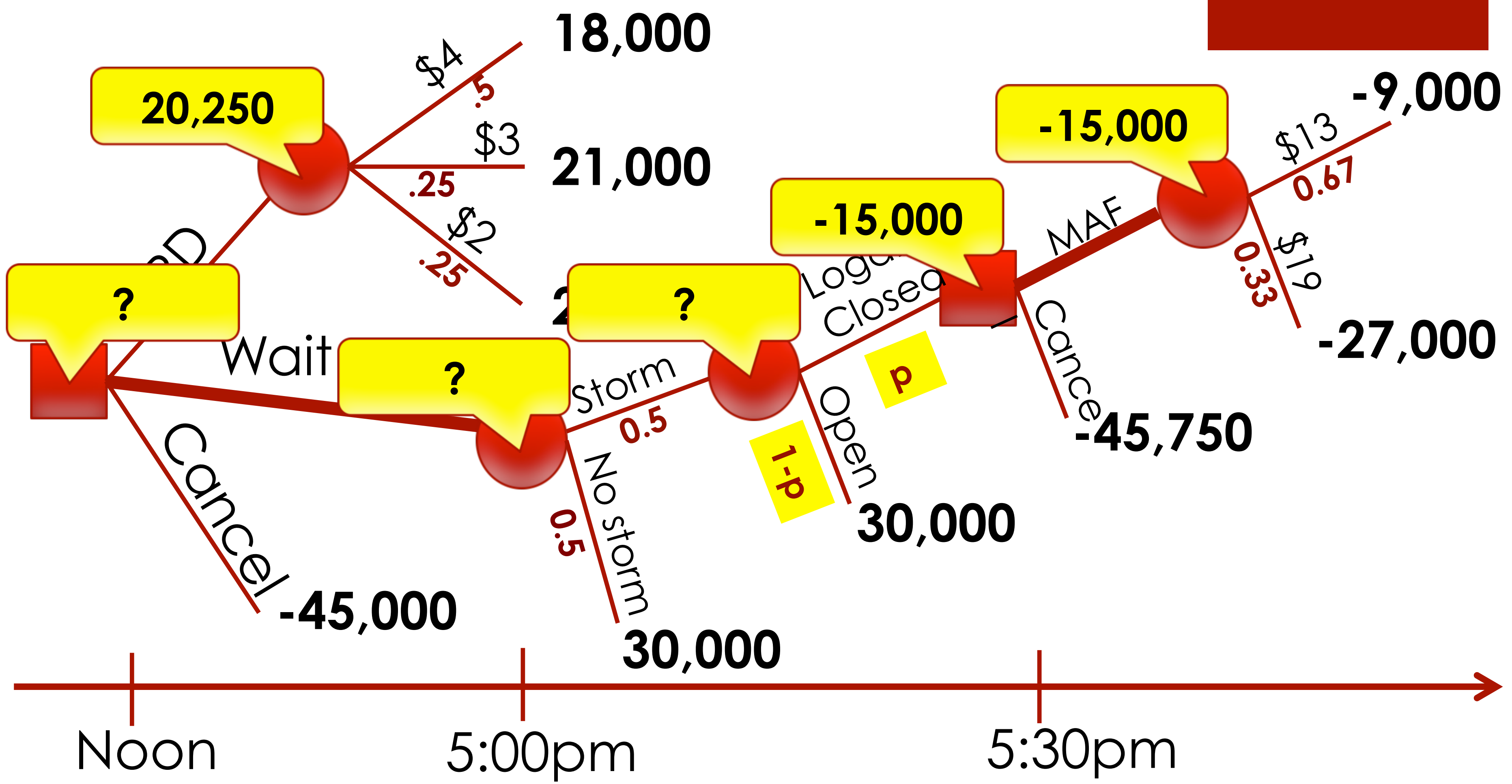

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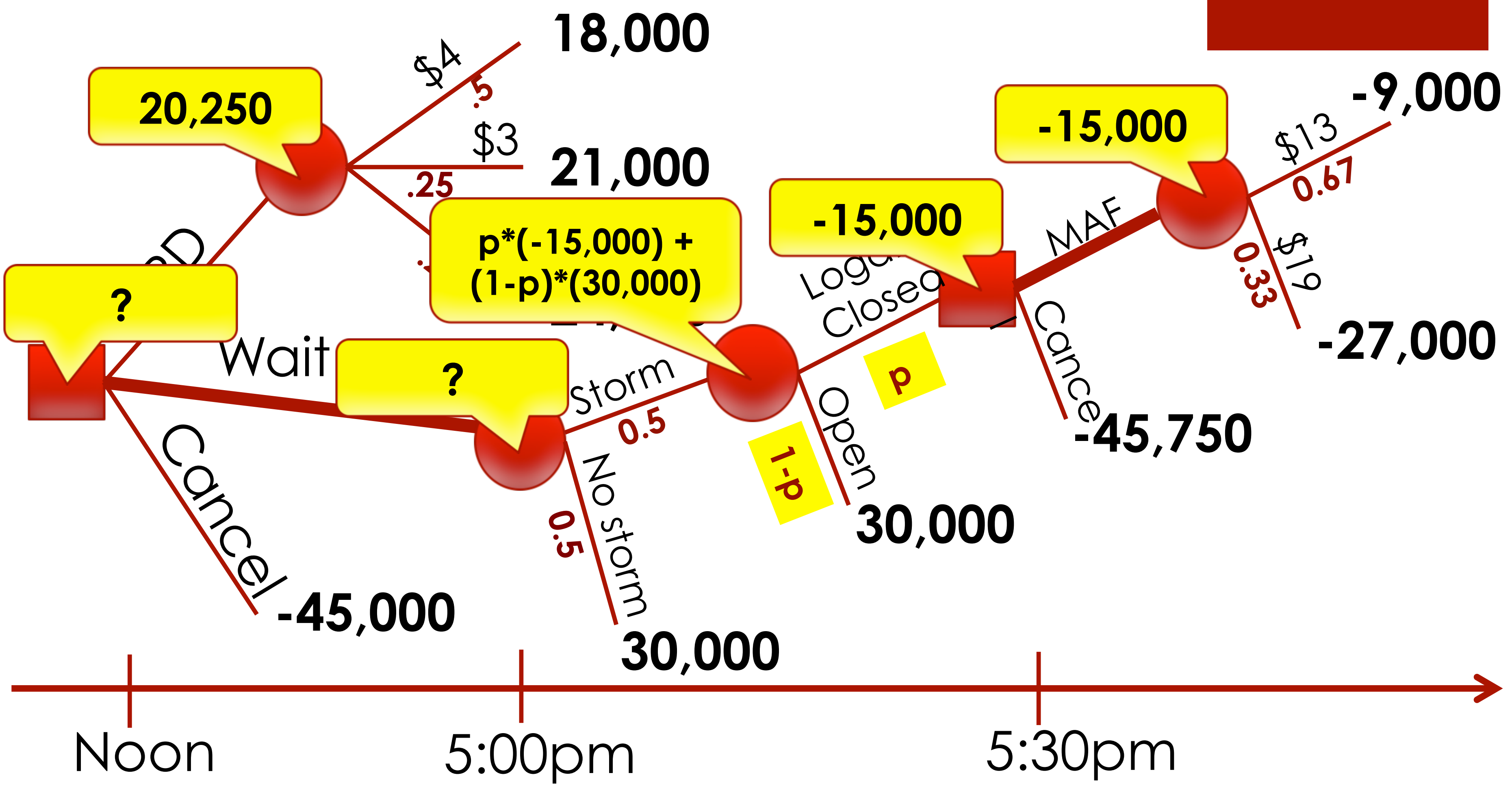

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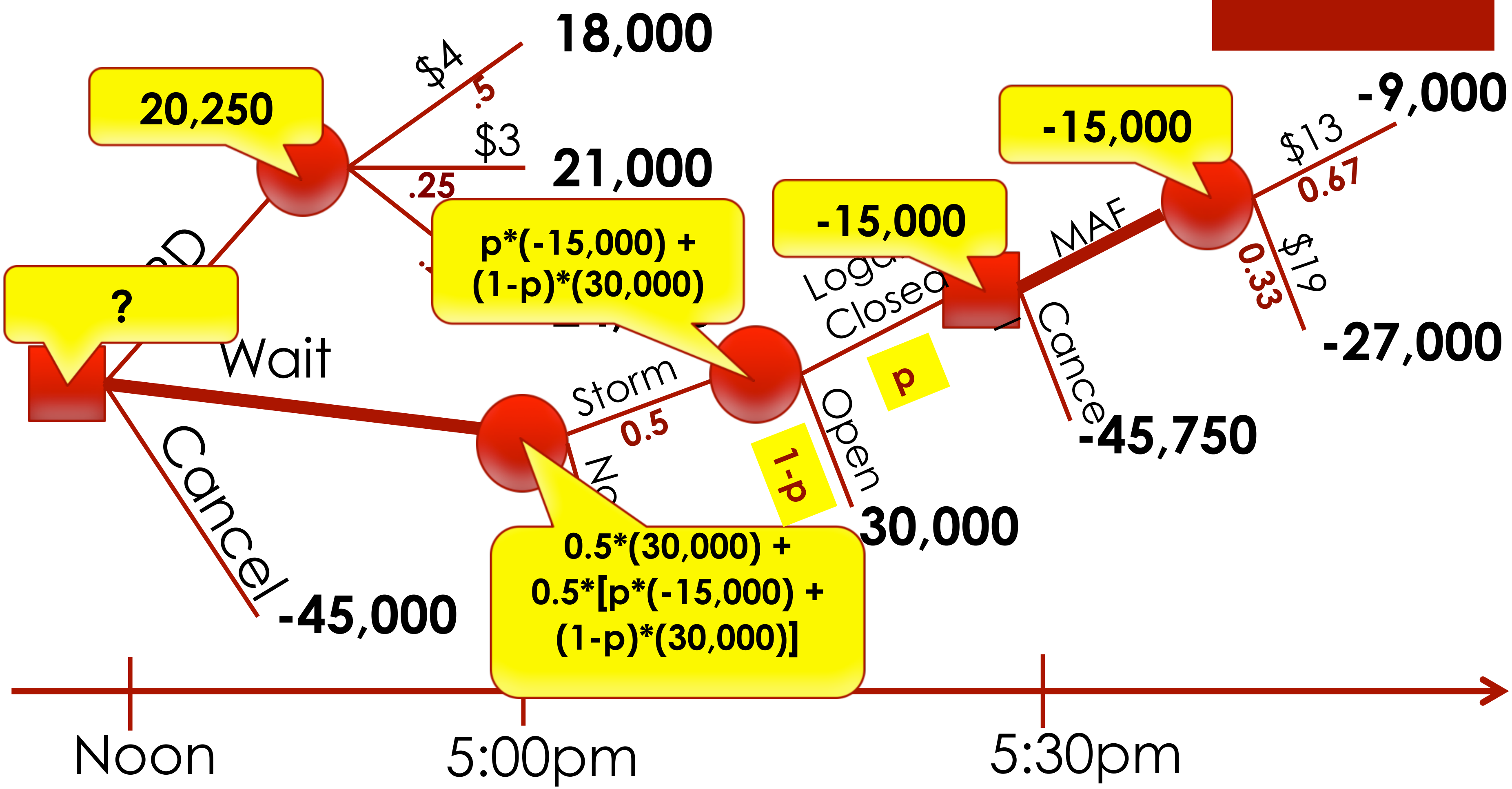
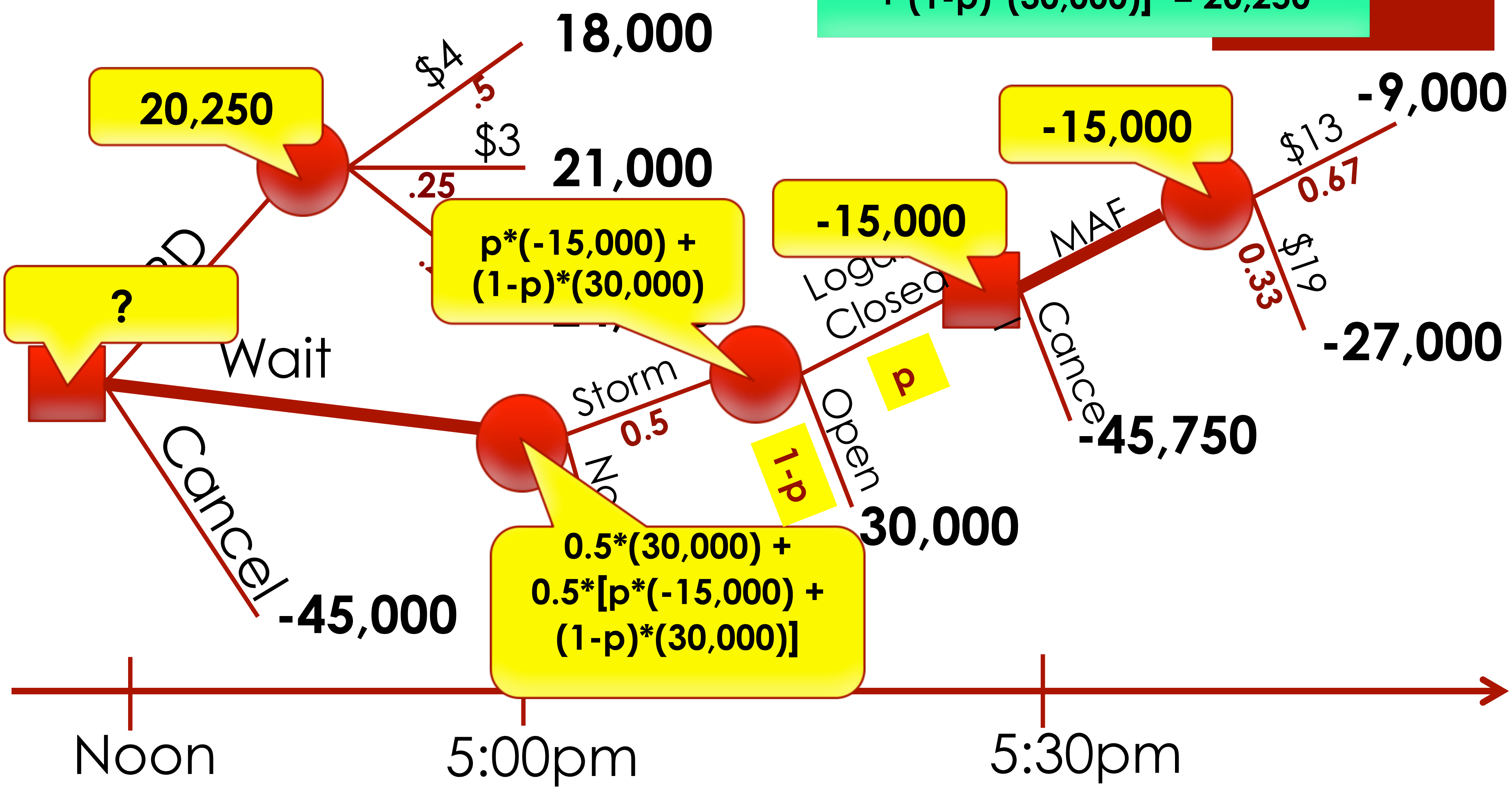


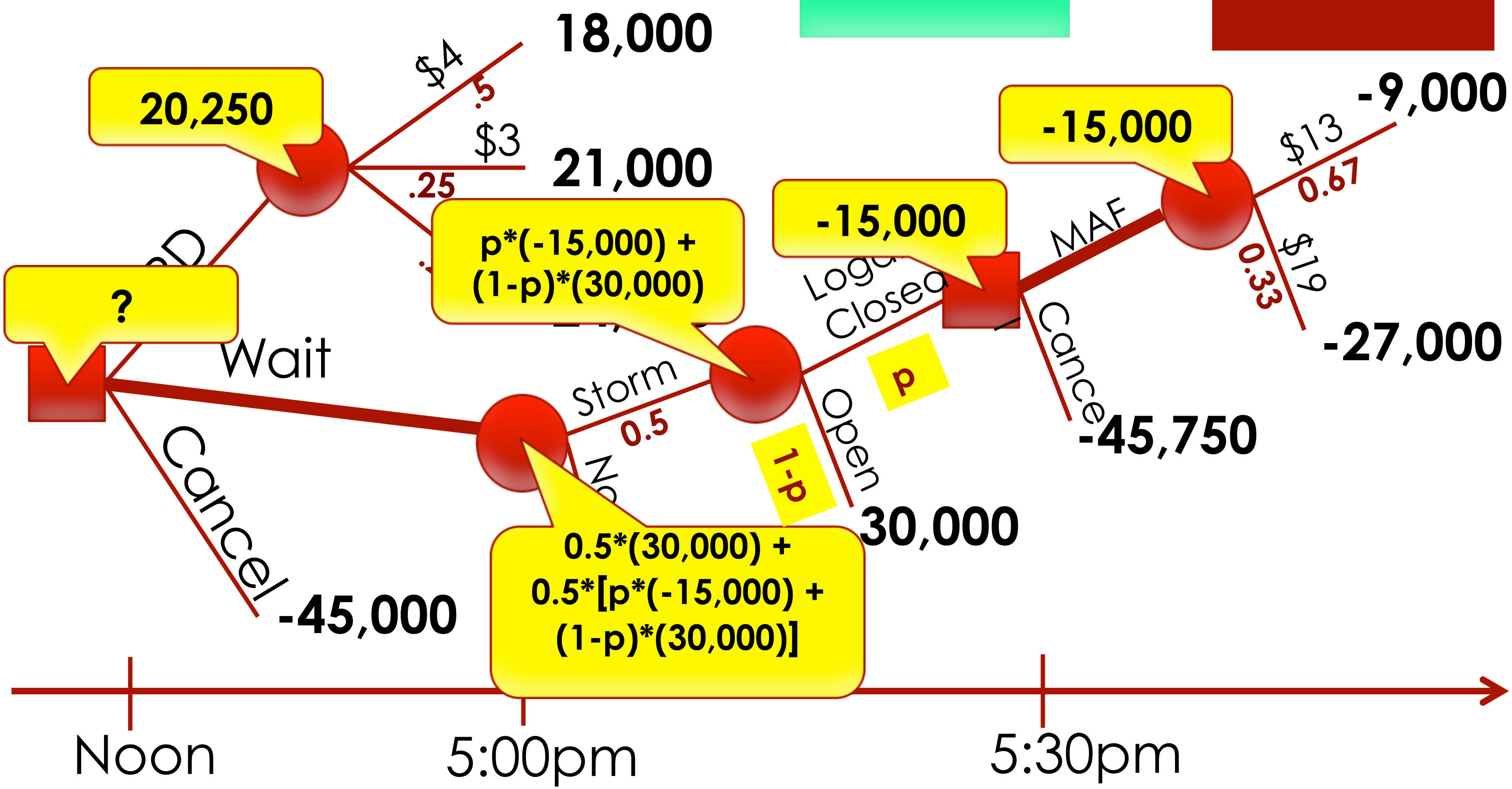
Sensitivity Analysis | Breakeven

0.5*(30,000) + 0.5*[p*(-15,000) + (1-p)*(30,000)] = 20,250



Sensitivity Analysis | Breakeven

p = 0.433



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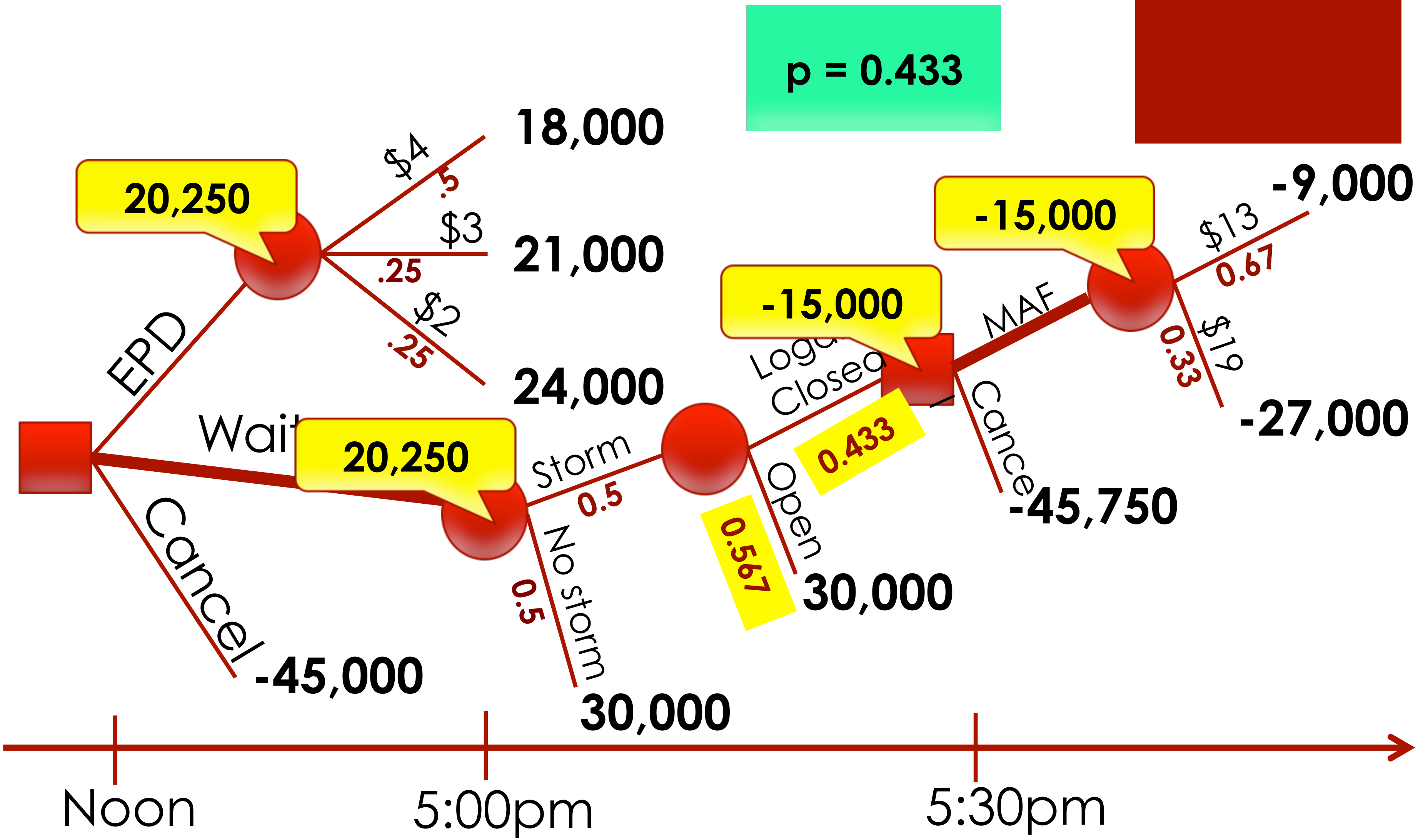

Sensitivity Analysis | Breakeven

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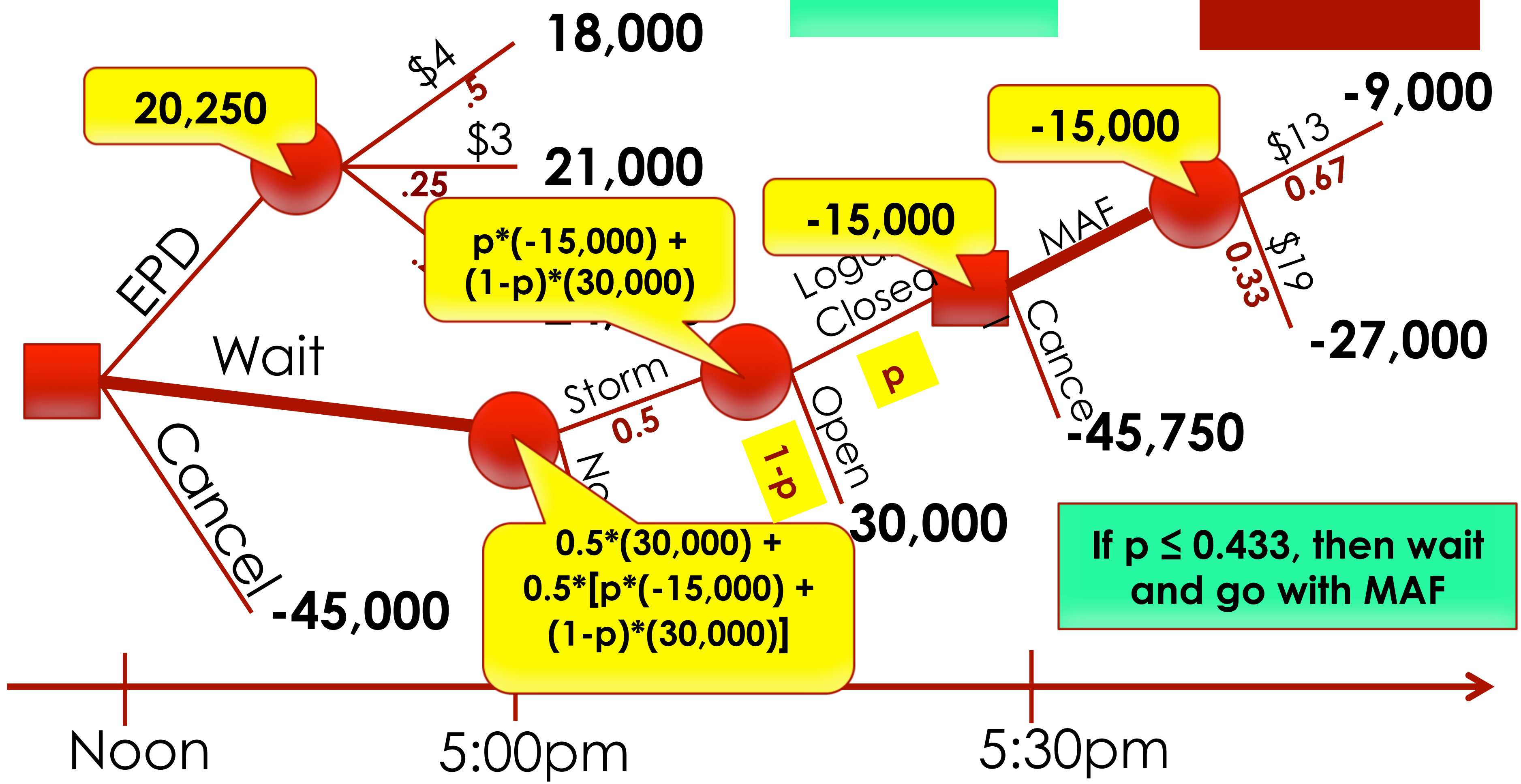
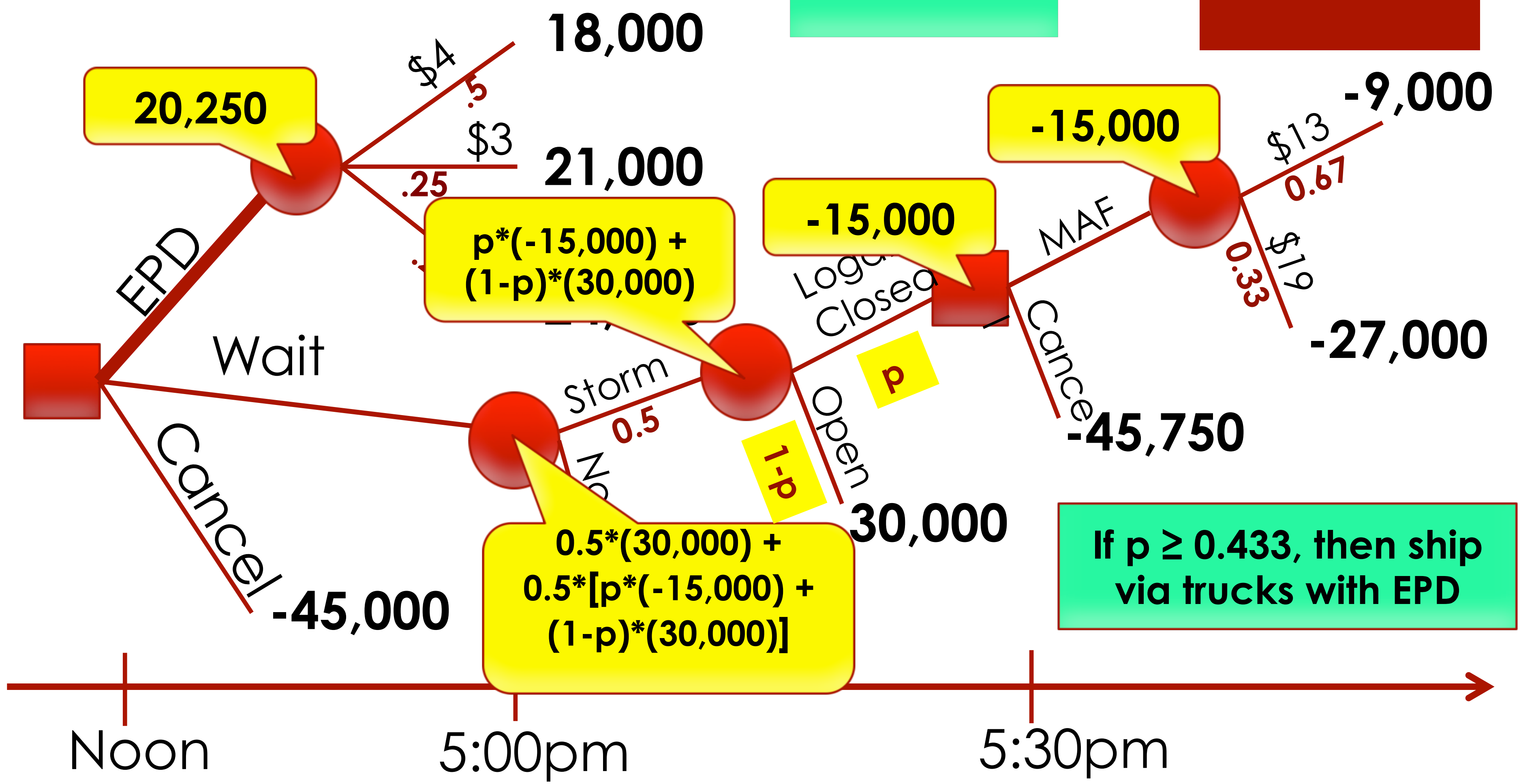


If p ≤ 0.433, then wait and go with MAF

Sensitivity Analysis | Breakeven

p = 0.433



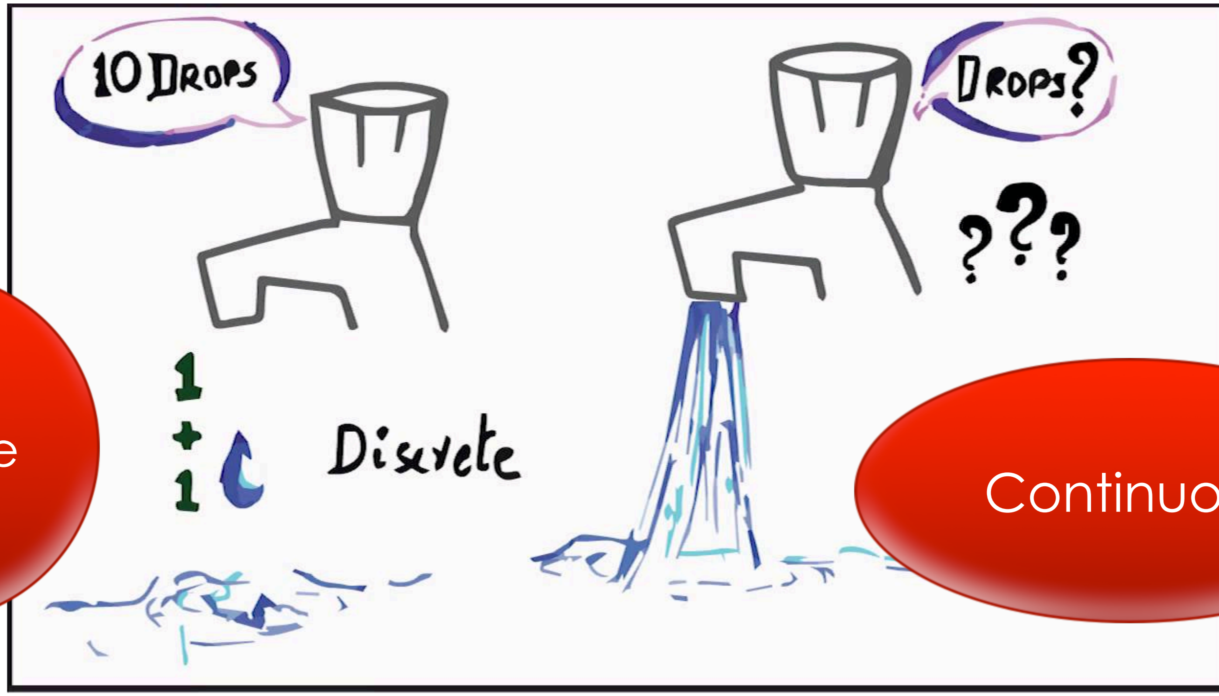
If p ≥ 0.433, then ship via trucks with EPD

Result of Decision Analysis

- An optimal decision rule (for every possible scenario): a "complete contingency plan"

- Insights:
  - If risk neutral, then computing EMV is good enough
  - But what if risk averse?
    - Maybe we want to be on the safe side and just ship by truck to DC!
    - Variability might be important to look at

Random Variables



Discrete

Continuous

Probability and Random Variables

- Remember!
  - Probabilities are always between 0 and 1
  - Considering all possible outcomes, the sum of their probabilities must add to ONE

R (% per year)	Probability
10	0.22
11	0.23
12	0.25
13	0.21
14	0.09

Complements

- What is the probability that return is at least 12%

  $P(R \geq 12) = P(R=12) + P(R=13) + P(R=14) = 0.25 + 0.21 + 0.09 = 0.55$

- What is the probability that return is less than 12%?

  $P(R < 12) = P(R=10) + P(R=11) = 0.22 + 0.23 = 0.45$

R (% per year)	Probability
10	0.22
11	0.23
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R (% per year)	Probability
10	0.22
11	0.23
12	0.25
13	0.21
14	0.09

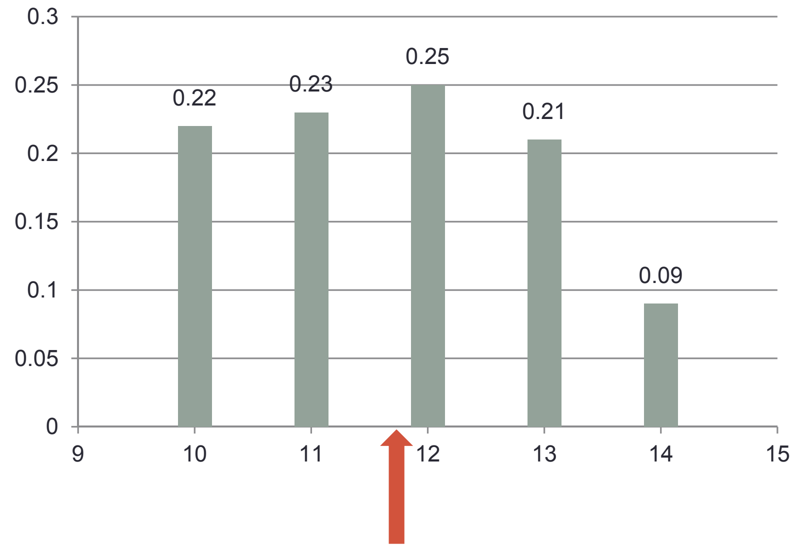
0.55 + 0.45 = 1

Expected Value

μ_R = 10*0.22 + 11*0.23 + 12*0.25 + 13*0.21 + 14*0.09 = 11.72
(Excel tip: SUMPRODUCT(column p_i, column r_i))

Interpretation: center of gravity

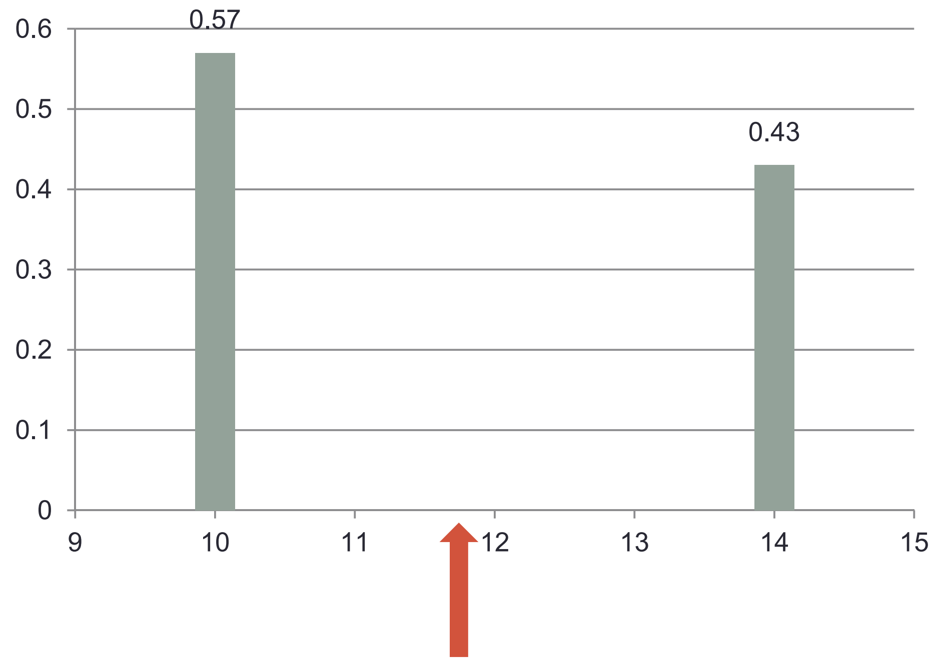
R (% per year)	Probability
10	0.22
11	0.23
12	0.25
13	0.21
14	0.09



Distribution can be “balanced” at the mean

Expected Value Doesn't Tell the Whole Story!

S (% per year)	Probability
10	0.57
11	0
12	0
13	0
14	0.43

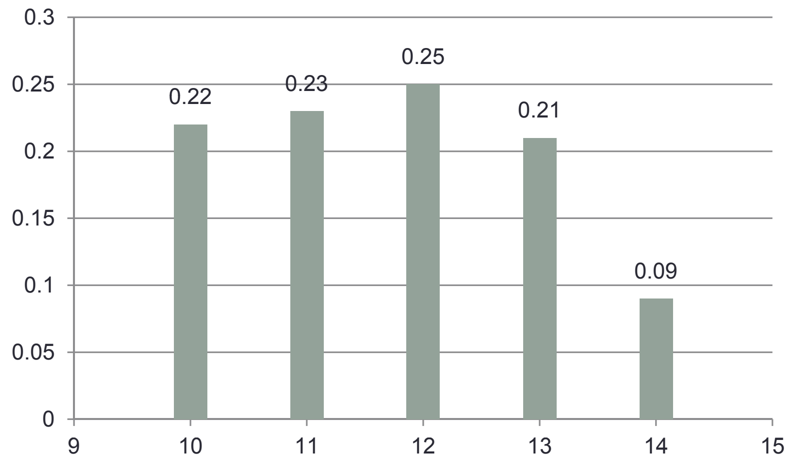


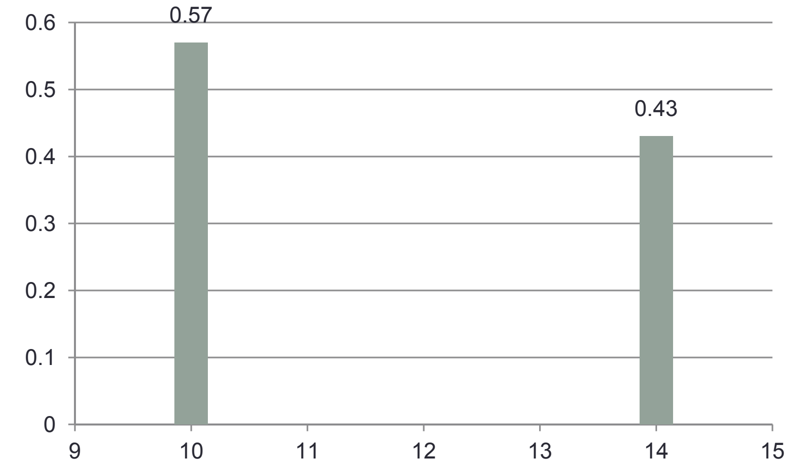
Distribution still balances at 11.72

The two portfolios have the same average return:

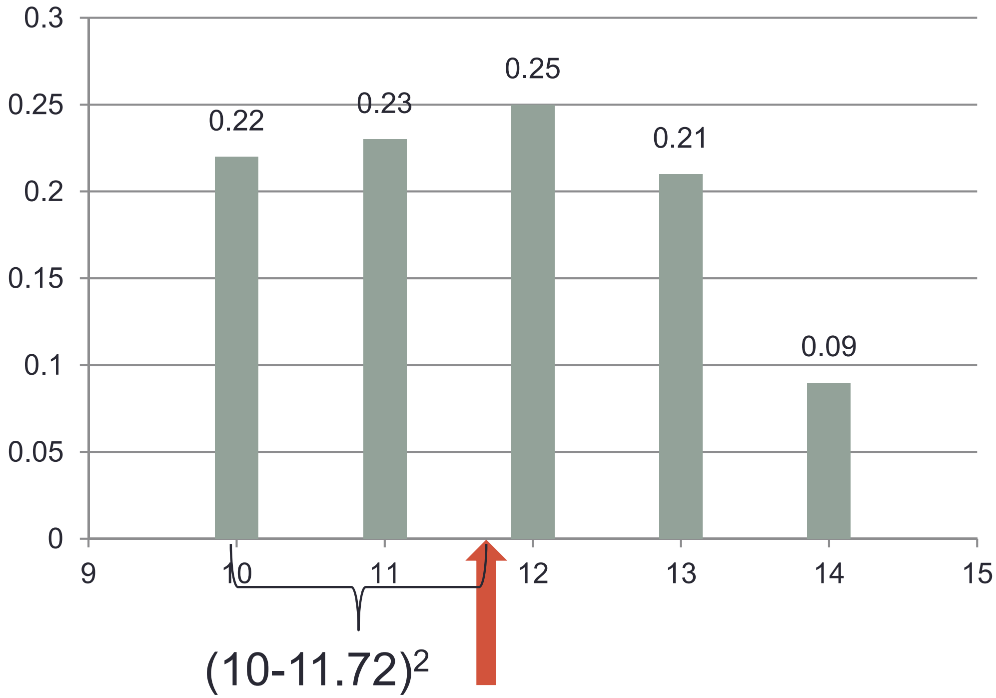
$\mu_S = 10*0.57 + 14*0.43 = 11.72 = \mu_R$

How are they different?



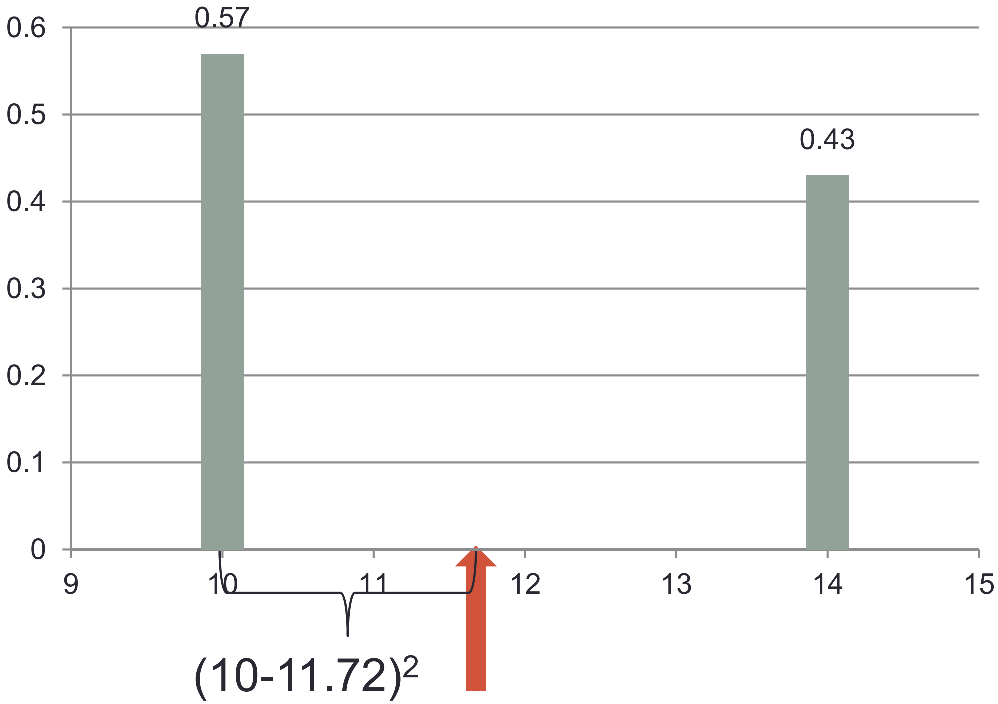


Variance



$$\sigma_R^2 = (10-11.72)^2 * (0.22)$$
$$+ (11-11.72)^2 * (0.23)$$
$$+ (12-11.72)^2 * (0.25)$$
$$+ (13-11.72)^2 * (0.21)$$
$$+ (14-11.72)^2 * (0.09)$$
$$= 1.6016$$

Variance



$$\sigma_S^2 = (10-11.72)^2 * (0.57)$$
$$+ (14-11.72)^2 * (0.43)$$
$$= 3.9216$$
$$> 1.6016 = \sigma_R^2$$

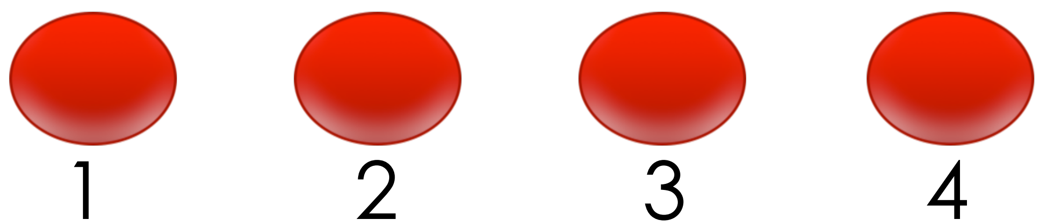
Portfolio S has more variability in its potential return than portfolio R

Binomial Distribution

- Describes the distribution of
  - the **number of successes**
  - out of **n independent "trials"**
  - where each trial has the **same probability of success p**
- Binomial(n,p)
- Need to properly define what "trial" and "success" mean for our problem!

Binomial Distribution

- Suppose we flip 4 coins → n=4
- Success is obtaining a head with p = 1/3
- X = number of heads



Binomial Distribution

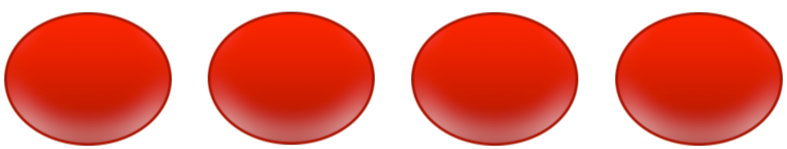
- How can we obtain 2 heads?

1	2	3	4	1	2	3	4
H	H	T	T	T	T	H	H
H	T	H	T	T	H	T	H
H	T	T	H	T	H	H	T

$$\frac{4!}{2!(4-2)!} = \frac{4*3*2*1}{2*1*(2*1)} = $$ 6 ways we can obtain 2 heads!

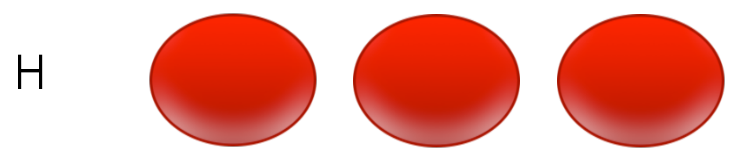
Binomial Distribution

- How can we obtain 2 heads?
- We can think of it a little differently!
  - Total number of permutations
    - To place the first head, we have 4 possibilities



Binomial Distribution

- How can we obtain 2 heads?
- We can think of it a little differently!
  - Total number of permutations
    - To place the first head, we have 4 possibilities
    - To place the second head, we have 3 possibilities



Binomial Distribution

- How can we obtain 2 heads?
- We can think of it a little differently!
  - Total number of permutations
    - To place the first head, we have 4 possibilities
    - To place the second head, we have 3 possibilities
    - To place the first tail, we have 2 possibilities

Binomial Distribution

- How can we obtain 2 heads?
- We can think of it a little differently!
  - Total number of permutations
    - To place the first head, we have 4 possibilities
    - To place the second head, we have 3 possibilities
    - To place the first tail, we have 2 possibilities
    - To place the second tail, we have 1 possibility

Binomial Distribution

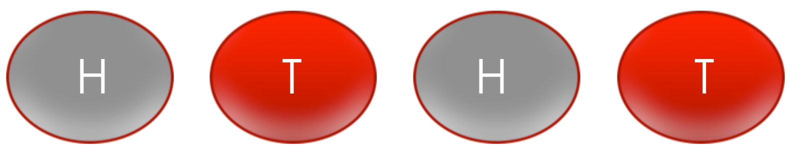
- How can we obtain 2 heads?
- We can think of it a little differently!
  - Total number of permutations
    - To place the first head, we have 4 possibilities
    - To place the second head, we have 3 possibilities
    - To place the first tail, we have 2 possibilities
    - To place the second tail, we have 1 possibility

4*3*2*1 = 4!

H T H T

Binomial Distribution

- How can we obtain 2 heads?
- We can think of it a little differently!
  - Total number of permutations = **4!**
  - But we don't care about the order of the heads or the tails!
    - Suppose we choose the following break-down



    - In how many ways can I place the heads in the grey bins?

Binomial Distribution

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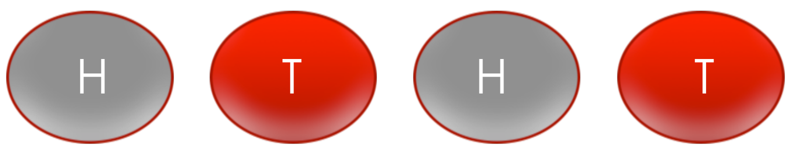

    - In how many ways can I place the heads in the grey bins? **2!**
    - In how many ways can I place the tails in the red bins?

Binomial Distribution

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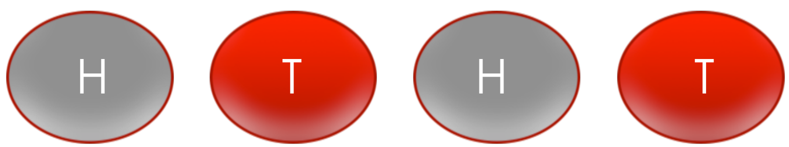

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Binomial Distribution

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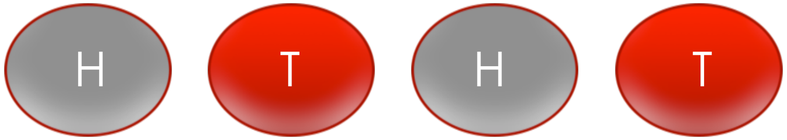


    - In how many ways can I place the heads in the grey bins? **2!**
    - In how many ways can I place the tails in the red bins? **(4-2)!**
  - Since we do not care about the order: **4!/(2! * 2!)**

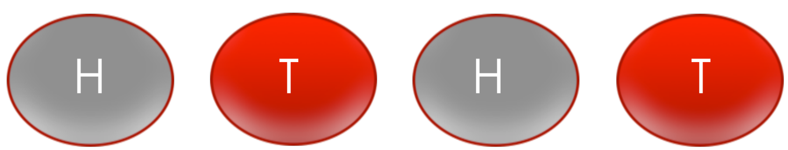
Binomial Distribution

- P(X=2) = probability that we obtain 2 heads and the remaining are tails
  - Each grey bin has a probability p=1/3 to have a head

$1/3 * 1/3 = (1/3)^2$

  - Each red bin has a probability 1-p = 2/3 to have a tail

$2/3 * 2/3 = (2/3)^{4-2}$



$$\rightarrow \frac{4!}{2!(4-2)!}(1/3)^2 * (2/3)^{4-2}$$

Binomial Distribution

- In general, if X is Binomial(n,p)
  - X can take on only the values 0,1,…,n-1,n
  - The probability that we have x successes out of n trials

$$\frac{n!}{x!(n-x)!} * (p)^x * (1-p)^{n-x}$$

Number of combinations with x successes out of n trial

Probability that x trials are successes

Probability that the remaining (n-k) trials are failures

Binomial RVs

- Expected value

$$E[X] = np$$

- Variance

$$Var(X) = np(1-p)$$

Revisiting Taylor Swift

Revisiting Taylor Swift

$$E(X) = 254{,}500$$
$$Std(X) = 503$$
$$CV(X) = Std(X)/E(X) = 0.0019$$

Revisiting Taylor Swift

$n = 50.9M, p = 0.005$

$E(X) = 254{,}500$

$Std(X) = 503$

$CV(X) = Std(X)/E(X) = 0.0019$

Same p: $E(X) = 0.67$
$Std(X) = 0.816$
$CV(X) = 1.219$

Revisiting Taylor Swift

n = 50.9M, p = 0.005

E(X) = 254,500

Std(X) = 503

CV(X) = Std(X)/E(X) = 0.0019

P = 0.0005: E(X) = 0.0067

Std(X) = 0.258

CV(X) = 3.862

Binomial Distribution - Example

- United's first class cabin has 10 seats in each plane.
- Overbooking policy is to sell up to 11 first class tickets since cancellations and no-shows are always possible
- Suppose that for a given flight
  - 11 first class tickets are sold
  - Each passenger has 80% chance of showing up for the flight
  - Whether a passenger shows up is independent of other passengers
- Can we model this as a binomial distribution?

Binomial Distribution - Example

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  - 11 first class tickets are sold
  - Each passenger has 80% chance of showing up for the flight
  - Whether a passenger shows up is independent of other passengers
- Can we model this as a binomial distribution? **YES!**

Binomial Distribution - Example

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- Overbooking policy is to sell up to 11 first class tickets since cancellations and no-shows are always possible

- Suppose that for a given flight
  - 11 first class tickets are sold
  - Each passenger has 80% chance of showing up for the flight
  - Whether a passenger shows up is independent of other passengers

- Trial = Passengers, Success = Showing up ⇨ $n = 11,\ p = 0.8$

Binomial Distribution - Example

- X = number of passengers that show up
- X is Binomial(11, 0.8)

- What is the probability that 10 passengers show up?

Binomial Distribution - Example

- X = number of passengers that show up
- X is Binomial(11, 0.8)
- What is the probability that 10 passengers show up?

$$P(X=10) = \frac{11!}{10!(11-10)!} (0.8)^{10} * (1-0.8)^{11-10}$$

$$= 11 * (0.8)^{10} * 0.2 \sim 0.236$$

Binomial Distribution - Example

- X = number of passengers that show up
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- What is the probability that the airline gets away with booking?

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$$P(X<=10)$$

Binomial Distribution - Example

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- X is Binomial(11, 0.8)
- What is the probability that the airline gets away with booking?

$$P(X<=10) = P(X=0) + P(X=1) + \ldots + P(X=10)$$

Binomial Distribution - Example

- X = number of passengers that show up
- X is Binomial(11, 0.8)
- What is the probability that the airline gets away with booking?

$$P(X<=10) = P(X=0) + P(X=1) + \ldots + P(X=10)$$

  - Or alternatively:

$$P(X<=10) = 1 - P(X>10) = 1 - P(X=11)$$

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$$P(X<=10) = 1 - P(X>10) = 1 - P(X=11)$$
$$= 1 - \frac{11!}{11!(11-11)!} 0.8^{11} * (1-0.8)^{11-11}$$

Binomial Distribution - Example

- X = number of passengers that show up
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$$= 1 - \frac{11!}{11!(11-11)!} 0.8^{11} * (1-0.8)^{11-11}$$

$$\sim 0.914$$

Binomial Distribution - Example

- X = number of passengers that show up
- X is Binomial(11, 0.8)

- What if the airline overbooked by selling
  - 11 seats: $P(X<=10) \sim 0.914$
  - 12 seats: $P(X<=10) \sim 0.725$
  - 13 seats: $P(X<=10) \sim 0.498$

Extensions

- Given some additional data
  - Fares prices
  - Cost of too many passengers showing up (refunds, damage to customer relations, etc.)
- Is it worthwhile to overbook flights?
- Check our assumptions
  - Is 80% an accurate probability?
  - Are passengers really independent?

Alternate Model

- We can define the "success" to be a passenger not showing up
- Y = number of passengers not showing up is Binomial(11,0.2)
- P(X=10) = P(exactly 10 passengers show up)

  = P(exactly 1 passenger does not show up)

  = P(Y=1) = $\dfrac{11!}{1!(11-1)!} 0.2^{1} * (1-0.2)^{11-1}$

Wrap-up

- Due electronically on Monday February 2, 2015
  - AOG case – work with your team!
  - Exercises 2.13 and 2.30 – complete individually!
  - Submit the PDF files on Stellar.
- Google Doc for Teams: bit.ly/DMD16-Teams
- Office hours for 30 minutes!
- Feel free to raise your hand and unmute yourself to ask questions!

Office Hour